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Proof Shortening: What's the Point?

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1. A Burning Question and a Surprising Answer

For many years, my research has been heavily focused on proof shortening, on taking a proof in hand with the object of finding one that is shorter when measured in deduced steps. An obvious question that you might ask—if not constrained by the bonds of politeness—is the following. What is the point? Indeed, why study proof shortening; why develop methodology designed to find proofs shorter than offered by the literature or by some researcher? This notebook features my answer to this more-than-reasonable question. Further, here I provide evidence, in the form of anecdotes, that my answer may be correct, that indeed various methodologies formulated for proof shortening in fact can be and have been successfully used for finding first proofs.

By way of amplification, I have developed over many years various methodologies (which I list soon) for finding a proof shorter than that in hand. The diverse approaches, when heavily relied upon, often do enable a program (such as W. McCune's OTTER) to discover new and shorter proofs. Each set of experiments begins with some given proof to shorten. The thesis in this notebook is that these same methodologies can often be applied to find a so-called first proof, to settle a conjecture, and to answer an open question. Each of the approaches perturbs the search space of conclusions to be examined by the automated reasoning program in use, sometimes sharply perturbs it. Apparently, both for proof shortening and for first-proof finding, such perturbing is often the key to success. Among the methodologies, I employ *resonators* to guide the reasoning, demodulation and weighting to guide and restrict the reasoning, *cramming* to attempt to force chosen formulas or equations to be used heavily, and various combinations of settings and assigned values to parameters. At the top level, iteration is frequently a key.

In the beginning (in the early 1960s), the field, now called automated reasoning by many researchers, was concerned with proving theorems. The main goal, I think it fair to say, was to add to mathematics and logic. The emphasis placed on answering open questions did not occur until the late 1970s.

The joy, excitement, and utter satisfaction resulting from answering an open question are indeed substantial. Understandably, you might believe that much expertise in the chosen area is required. With a powerful automated reasoning program and appropriate methodology, such is not the case. You may naturally ask about the nature of the methodology—whether it exists—and ask about an appropriate reasoning program. As for the latter, I (as expected by so many) recommend W. McCune's program OTTER. As for the former, this notebook offers a start, focusing on approaches that have answered open questions. (As an aside that has provided amusement—or horror—for some of my colleagues, I have answered various open questions even though, upon being interrogated, I noted that I had no idea about the field from which the

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question was taken.)

The various approaches come from studies designed to seek proofs shorter than in hand. You thus have your answer to the question posed earlier, a most reasonable question indeed. What is the point of a heavy emphasis on proof shortening, or, for that matter, why ever seek shorter proofs? I shall first present an example, actually a story, of a case of proof shortening that led to what might be termed an accidental discovery, an answer to an open question posed by the logician D. (Ted) Ulrich. With more stories, I shall then offer evidence that, when the goal was that of explicitly answering various open questions, the use of methodology designed for proof shortening led to treasure.

2. A Story of Accidental Discovery

This notebook exists because of two people, namely, John Halleck and Ross Overbeek. The former provided the wellspring, in the form of a 70-step proof (obtained with his program shotgun), for the studies reported here. The latter provided the encouragement—actually, almost a demand—for me to capture on paper what occurred. The cited proof focuses on the *BCI* logic that is frequently axiomatized with the following three formulas, expressed as clauses, where the function *i* denotes implication.

```
P(i(i(x,y),i(i(z,x),i(z,y)))).  %  B, in infix  (x->y) -> ((z->x)->(z->y))
P(i(i(x,i(y,z)),i(y,i(x,z)))).  %  C, in infix  (x->(y->z)) -> (y->(x->z))
P(i(x,x)).  %  I, in infix x -> x
```

At the time the study commenced, twenty-eight shortest single axioms were in hand, two from C. A. Meredith and twenty-six from D. Ulrich. (Throughout this notebook, the phrase "single axiom" always refers to a shortest single axiom.) The 70-step proof was obtained by Halleck with his own program when he sought a short proof that deduced, from *B*, *C*, and *I*, all twenty-eight known (at the time) single axioms. Knowing of my successes with finding proofs shorter than those in hand, he asked me (I believe in February 2009) to find a proof of length less than seventy, measured in terms of deduced steps. The inference rule to be used was condensed detachment, captured with the following clause in the presence of hyperresolution, where "-" denotes logical **not** and " | " denotes logical **or**.

```
-P(i(x,y)) | -P(x) | P(y).
```

I did find the proof that Halleck requested—actually, a number of proofs. As required, each of the proofs contains, among its derived steps, all twenty-eight known (at the time) single axioms. Among the proofs was one that offered the following formula for strong consideration as a possible new single axiom for the *BCI* logic, whose story is told in a forthcoming notebook entitled bci.revisited, a notebook that will be found on my website automatedreasoning.net.

```
P(i(i(i(x,x),i(y,i(z,u))),i(i(u,v),i(z,i(y,v))))).  %  formula 29
```

The cited formula was particularly appealing in that its length is identical to the length of each of the twenty-eight target single axioms, and, further (as is the case for all twenty-eight) the formula relies on precisely five distinct variables. Especially intriguing was the fact that, other than the twenty-eight known (at the time) single axioms and formula 29, no other derived step had the given length and distinct-variable property. I cite this as an "accidental discovery" because I in no way was seeking anything other than the sought-after short proof. Halleck immediately with his program pounced on formula 29, succeeding in proving it to be a new single axiom, one among fifty-seven offered by Ulrich as possible new single axioms for the *BCI* logic. (At the time, Halleck and I were unaware of the list of fifty-seven.) So you have a nice example of how proof shortening indirectly led to the discovery of a new single axiom. The discovery that formula 29 is a single axiom resulted in Ulrich contributing (with his program) forty-three additional new single axioms. Equally important, and of especial relevance to this notebook, Halleck then found yet another new single axiom, formula 18, the following.

```
P(i(i(x,y),i(i(i(i(i(z,z),i(y,u)),u),x2),i(x,x2)))).  % BCI candidate 18
```

As you will immediately see, when the notebook now turns to the use of methodology originally formulated for proof shortening but used here explicitly for finding first proofs, Halleck's success with formula 18 led me in a new research direction. Further, you will see that his proof, establishing candidate 18 to be a

single axiom for *BCI*, played a key role.

3. Deliberate Treasure Hunting

Motivated by the Halleck success, I decided to try my hand at finding a new shortest single axiom for *BCI*. How might I proceed? After all, my expertise at this point in my research resided far more with proof shortening than with first-proof finding. The obvious approach to me was to simply proceed as I often do when seeking a proof shorter than I had in hand.

I did have the Halleck 42-step proof for candidate 18, deducing the conjunction or join of *B*, *C*, and *I*. I also had a 23-step proof I had obtained when seeking a short proof that formula 29 implies the cited join. When I seek a shorter proof, I often turn to various proofs that are closely or distantly related to the proof in hand, using their deduced steps sometimes as *resonators* (in this case), sometimes as hints, to guide the reasoning of the program. Resonators are formulas or equations that do not take on a **true** or **false** value; instead, their functional shape is the key, treating all variables as indistinguishable. Their presence strongly influences the direction the proof search takes, having the program prefer any item that matches an included resonator for inference-rule initiation. Therefore, as I would if the goal were proof shortening, I chose to key on the deduced steps of the two proofs. The basic idea is to first of all guide the search with earlier successes and second to possibly introduce new formulas into a possible proof. I cannot say how I chose the formula to study, but candidate 23 (the following) from Ulrich was the focus, a formula whose status with regard to being a single axiom was open. (There remained at the time eleven such formulas from Ulrich; Halleck had conquered candidate 18 and candidate 10; the list of thirteen that had been open is offered in the bic.revisited notebook.)

P(i(i(u,v),i(i(i(i(w,x),x),i(i(y,y),u)),i(w,v)))). % BCI-Candidate 23

The following input file sufficed to produce the desired proof.

An Input File for Studying Candidate 23

```
set(hyper_res).
assign(max_weight,48).
%  assign(change_limit_after,800).
%  assign(new_max_weight,22).
assign(max_proofs,-1).
clear(print_kept).
%  set(ancestor_subsume).
set(back_sub).
% clear(for_sub).
clear(print_back_sub).
clear(print_kept).
clear(print_new_demod).
clear(print_back_demod).
clear(print_back_sub).
assign(max_distinct_vars,14).
assign(pick_given_ratio,2).
assign(max_mem,750000).
%  assign(max_seconds,2).
assign(report,5400).
set(order_history).
set(input_sos_first).
%  set(sos_queue).
assign(bsub_hint_wt,1).
set(keep_hint_subsumers).
```

```
weight_list(pick_and_purge).
%  Following 42 apparently prove candidate 18 a new single axiom, from Halleck.
weight(P(i(i(x,y),i(i(i(i(i(z,z),i(y,u)),u),x2),i(x,x2)))),-4).  % BCI candidate 18
weight(P(i(i(i(i(i(x,x),i(i(i(i(i(y,y),i(z,u)),u),x1),i(u,x1)),v)),v),w),i(i(u,z),w))),-4).
weight(P(i(i(x,i(y,z)),i(i(u,y),i(u,i(x,z))))),-4).
weight(P(i(i(i(i(i(x,x),i(i(i(y,z),u),x2)),x2),u),i(i(i(i(i(v,v),i(i(i(i(i(w,w),i(z,x)),x),y),
  i(y,y)),z)),z),u),u))),-4).
weight(P(i(i(i(i(i(x,x),i(i(i(i(i(y,y),i(z,u)),u),x2),i(u,x2)),v)),v),w),i(i(x,y),i(x,i(i(i(z,z),
  i(i(i(u,z),w),i(y,x1))),x1))))),-4).
weight(P(i(i(x,y),i(x,i(i(i(z,z),i(i(i(u,i(x2,u)),i(i(v,x2),i(v,i(u,u)))),i(y,w))),w)))),-4).
weight(P(i(i(x,i(i(i(i(y,y),i(z,u)),u),x2)),i(x,i(i(u,z),i(u,x2))))),-4).
weight(P(i(i(x,i(y,z)),i(i(u,x2),i(u,i(i(i(u,u),i(x2,y)),i(x,z)))))),-4).
weight(P(i(i(x,y),i(x,i(i(i(z,z),i(y,i(i(i(i(u,u),i(x2,u)),u),v))),i(i(w,x2),i(w,v)))))),-4).
weight(P(i(i(x,i(y,z)),i(x,i(i(u,i(z,x2)),i(y,i(u,x2)))))),-4).
weight(P(i(i(x,i(y,z)),i(i(u,i(i(i(x2,i(z,u)),i(y,i(x2,u))),v)),i(x,i(u,v))))),-4).
weight(P(i(i(x,i(y,i(i(i(z,i(u,x2)),i(u,i(z,x2))),v))),i(x,i(i(w,i(u,u)),i(w,i(y,v)))))),-4).
weight(P(i(i(x,y),i(i(z,i(y,i(i(u,u),i(x2,u)))),i(z,i(x,i(i(v,x2),i(v,u)))))),-4).
weight(P(i(i(x,i(y,z)),i(i(u,i(i(x2,i(x,z)),u)),i(i(x2,y),i(u,u))))),-4).
weight(P(i(i(x,i(i(y,i(i(z,i(u,x2)),i(u,i(z,x2)))),v)),i(i(y,i(u,u)),i(x,v)))),-4).
weight(P(i(i(x,i(i(y,y),z)),i(i(u,i(x2,i(z,u))),i(x,i(u,i(i(v,x2),i(v,u)))))),-4).
weight(P(i(i(x,i(y,i(i(z,i(i(i(u,u),i(x2,u)),u)),v))),i(i(z,x2),i(x,i(i(w,y),i(w,v)))))),-4).
weight(P(i(i(i(x,x),i(y,z)),i(i(u,y),i(i(x2,u),i(x2,i(i(z,u),u)))))),-4).
weight(P(i(i(x,i(i(i(i(y,y),i(z,u)),u),x2)),i(i(u,x),i(u,i(i(i(z,x2),v),v))))),-4).
weight(P(i(i(x,i(y,i(z,u))),i(x,i(i(i(z,i(y,u)),x2),x2)))),-4).
weight(P(i(i(x,i(y,z)),i(i(i(u,i(i(u,y),i(x,z))),x2),x2))),-4).
weight(P(i(i(x,i(y,z)),i(i(u,i(i(x2,x2),y)),i(u,i(x,z))))),-4).
weight(P(i(i(i(i(i(x,x),i(i(i(y,z),i(y,i(u,x2))),u)),u),v),i(i(u,i(z,x2)),v))),-4).
weight(P(i(i(x,i(y,i(z,u))),i(i(i(i(i(z,i(y,u)),x2),i(x,x2)),u),u))),-4).
weight(P(i(i(i(i(i(x,i(i(x,y),i(z,u))),x2),i(i(z,i(y,u)),x2)),u),u)),-4).
weight(P(i(i(x,i(y,z)),i(i(u,i(i(y,i(x,z)),x2)),i(u,x2)))),-4).
weight(P(i(i(x,i(i(i(i(y,z),i(i(u,i(z,x2)),i(y,i(u,x2)))),u)),i(x,u))),-4).
weight(P(i(i(x,i(y,z)),i(i(u,x),i(u,i(y,z))))),-4).
weight(P(i(i(x,i(y,z)),i(i(i(u,i(i(u,x),i(y,z))),x2),x2))),-4).
weight(P(i(i(x,i(y,z)),i(i(u,i(i(x2,x2),x)),i(u,i(y,z))))),-4).
weight(P(i(i(i(x,i(i(y,z),z)),i(u,x2)),i(i(x,y),i(u,x2)))),-4).
weight(P(i(i(x,i(y,z)),i(i(i(u,u),i(x2,y)),i(x2,i(x,z))))),-4).
weight(P(i(i(x,i(i(y,i(z,u)),x2)),i(i(z,i(y,u)),i(x,x2)))),-4).
weight(P(i(i(i(x,x),i(y,z)),i(i(u,i(z,x2)),i(y,i(u,x2))))),-4).
weight(P(i(i(x,i(i(i(i(y,i(z,u)),x2),x2),u)),i(i(z,i(y,u)),i(x,u)))),-4).
weight(P(i(i(x,i(i(y,y),z)),i(i(u,x),i(u,z)))),-4).
weight(P(i(i(x,y),i(i(z,x),i(z,y)))),-4).  % B
weight(P(i(i(x,i(y,z)),i(x,i(y,z)))),-4).
weight(P(i(i(x,i(i(y,y),i(z,u))),i(x,i(z,u)))),-4).
weight(P(i(i(x,i(y,z)),i(y,i(x,z)))),-4).  % C
weight(P(i(i(x,y),i(x,y))),-4).
weight(P(i(i(i(x,x),i(y,z)),i(y,z))),-4).
weight(P(i(i(x,x),i(y,z))),-4).
%  Following 23/16 prove join of BCI from the 29th odd one, temp.bci.halleck.out2e.
weight(P(i(i(i(i(x,y),z),u),i(x,i(i(y,z),u)))),-4).
weight(P(i(i(i(i(x,y),z),u),i(i(y,z),i(x,u)))),-4).
weight(P(i(i(x,y),i(i(z,u),i(z,i(i(u,x),y))))),-4).
weight(P(i(i(i(i(x,y),y),z),i(u,i(i(u,x),z)))),-4).
weight(P(i(x,i(i(x,i(y,z)),i(u,i(i(u,y),z))))),-4).
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weight(P(i(x,i(i(x,i(y,z)),i(y,i(i(z,u),u))))),-4).
weight(P(i(i(i(i(x,y),y),z),i(u,i(i(i(v,v),i(u,x)),z)))),-4).
weight(P(i(i(i(i(x,y),y),z),i(i(i(u,u),i(v,x)),i(v,z)))),-4).
weight(P(i(i(i(x,x),i(y,i(z,u))),i(y,i(i(u,v),i(z,v))))),-4).
weight(P(i(i(i(x,x),i(y,z)),i(i(i(i(z,u),u),v),i(y,v)))),-4).
weight(P(i(i(i(x,x),i(y,z)),i(i(i(i(u,y),z),v),i(u,v)))),-4).
weight(P(i(i(x,y),i(i(i(i(y,z),z),u),i(x,u)))),-4).
weight(P(i(i(i(i(x,y),y),z),i(i(z,u),i(x,u)))),-4).
weight(P(i(i(i(i(x,i(i(i(y,z),z),u)),i(y,u)),v),i(x,v))),-4).
weight(P(i(i(i(i(i(x,x),i(y,z)),i(y,u)),v),i(i(z,u),v))),-4).
weight(P(i(i(x,i(i(i(i(i(y,z),z),u),u),v)),i(x,i(y,v)))),-4).
weight(P(i(i(x,y),i(i(z,x),i(z,y)))),-4).
weight(P(i(i(x,i(y,z)),i(y,i(x,z)))),-4).
weight(P(i(i(i(x,x),i(y,i(z,u))),i(z,i(y,u)))),-4).
weight(P(i(x,i(i(i(y,y),i(z,u)),i(i(x,z),u)))),-4).
weight(P(i(i(i(x,x),y),i(i(i(z,z),i(y,u)),u))),-4).
weight(P(i(i(i(x,x),i(i(y,i(i(y,z),z)),u)),u)),-4).
weight(P(i(x,x)),-4).
end_of_list.

list(usable).
-P(i(x,y)) | -P(x) | P(y).
-P(i(i(a1,i(b,a2)),i(b,i(a1,a2)))) | -P(i(a1,a1)) | -P(i(i(a1,b),i(i(a2,a1),i(a2,b)))) | $ANS(all).  % BCI
end_of_list.

list(sos).
P(i(i(u,v),i(i(i(i(w,x),x),i(i(y,y),u)),i(w,v)))). % BCI-Candidate 23
end_of_list.

list(passive).
%  Following negs of a purported 42-step proof showing candidate 18 to be a single axiom.
-P(i(i(i(i(i(a,a),i(i(i(i(i(b,b),i(c,d)),d),a1),i(d,a1)),e)),e),f),i(i(d,c),f))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(i(d,b),i(d,i(a,c))))) |
  $ANS(TARG4).
-P(i(i(i(i(i(a,a),i(i(i(b,c),d),a2)),a2),d),i(i(i(i(i(e,e),i(i(i(i(i(f,f),i(c,a)),a),b),
  i(b,b)),c)),c),d),d))) | $ANS(TARG4).
-P(i(i(i(i(i(a,a),i(i(i(i(i(b,b),i(c,d)),d),a2),i(d,a2)),e)),e),f),i(i(a,b),i(a,i(i(i(c,c),
  i(i(i(d,c),f),i(b,a1))),a1))))) | $ANS(TARG4).
-P(i(i(a,b),i(a,i(i(i(c,c),i(i(i(d,i(a2,d)),i(i(e,a2),i(e,i(d,d)))),i(b,f))),f)))) | $ANS(TARG4).
-P(i(i(a,i(i(i(i(b,b),i(c,d)),d),a2)),i(a,i(i(d,c),i(d,a2))))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(i(d,a2),i(d,i(i(i(d,d),i(a2,b)),i(a,c)))))) | $ANS(TARG4).
-P(i(i(a,b),i(a,i(i(i(c,c),i(b,i(i(i(i(d,d),i(a2,d)),d),e))),i(i(f,a2),i(f,e)))))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(a,i(i(d,i(c,a2)),i(b,i(d,a2)))))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(i(d,i(i(i(a2,i(c,d)),i(b,i(a2,d))),e)),i(a,i(d,e))))) | $ANS(TARG4).
-P(i(i(a,i(b,i(i(i(c,i(d,a2)),i(d,i(c,a2))),e))),i(a,i(i(f,i(d,d)),i(f,i(b,e)))))) | $ANS(TARG4).
-P(i(i(a,b),i(i(c,i(b,i(i(d,d),i(a2,d)))),i(c,i(a,i(i(e,a2),i(e,d))))))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(i(d,i(i(a2,i(a,c)),d)),i(i(a2,b),i(d,d))))) | $ANS(TARG4).
-P(i(i(a,i(i(b,i(i(c,i(d,a2)),i(d,i(c,a2)))),e)),i(i(b,i(d,d)),i(a,e)))) | $ANS(TARG4).
-P(i(i(a,i(i(b,b),c)),i(i(d,i(a2,i(c,d))),i(a,i(d,i(i(e,a2),i(e,d))))))) | $ANS(TARG4).
-P(i(i(a,i(b,i(i(c,i(i(i(d,d),i(a2,d)),d)),e))),i(i(c,a2),i(a,i(i(f,b),i(f,e)))))) | $ANS(TARG4).
-P(i(i(i(a,a),i(b,c)),i(i(d,b),i(i(a2,d),i(a2,i(i(c,d),d)))))) | $ANS(TARG4).
-P(i(i(a,i(i(i(i(b,b),i(c,d)),d),a2)),i(i(d,a),i(d,i(i(i(c,a2),e),e))))) | $ANS(TARG4).
-P(i(i(a,i(b,i(c,d))),i(a,i(i(i(c,i(b,d)),a2),a2)))) | $ANS(TARG4).
```

-P(i(i(a,i(b,c)),i(i(i(d,i(i(d,b),i(a,c))),a2),a2))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(i(d,i(i(a2,a2),b)),i(d,i(a,c))))) | $ANS(TARG4).
-P(i(i(i(i(i(a,a),i(i(i(b,c),i(b,i(d,a2))),d)),d),e),i(i(d,i(c,a2)),e))) | $ANS(TARG4).
-P(i(i(a,i(b,i(c,d))),i(i(i(i(i(c,i(b,d)),a2),i(a,a2)),d),d))) | $ANS(TARG4).
-P(i(i(i(i(i(a,i(i(a,b),i(c,d))),a2),i(i(c,i(b,d)),a2)),d),d)) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(i(d,i(i(b,i(a,c)),a2)),i(d,a2)))) | $ANS(TARG4).
-P(i(i(a,i(i(i(b,c),i(i(d,i(c,a2)),i(b,i(d,a2)))),d)),i(a,d))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(i(d,a),i(d,i(b,c))))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(i(i(d,i(i(d,a),i(b,c))),a2),a2))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(i(d,i(i(a2,a2),a)),i(d,i(b,c))))) | $ANS(TARG4).
-P(i(i(i(a,i(i(b,c),c)),i(d,a2)),i(i(a,b),i(d,a2)))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(i(i(d,d),i(a2,b)),i(a2,i(a,c))))) | $ANS(TARG4).
-P(i(i(a,i(i(b,i(c,d)),a2)),i(i(c,i(b,d)),i(a,a2)))) | $ANS(TARG4).
-P(i(i(i(a,a),i(b,c)),i(i(d,i(c,a2)),i(b,i(d,a2))))) | $ANS(TARG4).
-P(i(i(a,i(i(i(i(b,i(c,d)),a2),a2),d)),i(i(c,i(b,d)),i(a,d)))) | $ANS(TARG4).
-P(i(i(a,i(i(b,b),c)),i(i(d,a),i(d,c)))) | $ANS(TARG4).
-P(i(i(a,b),i(i(c,a),i(c,b)))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(a,i(b,c)))) | $ANS(TARG4).
-P(i(i(a,i(i(b,b),i(c,d))),i(a,i(c,d)))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(b,i(a,c)))) | $ANS(TARG4).
-P(i(i(a,b),i(a,b))) | $ANS(TARG4).
-P(i(i(i(a,a),i(b,c)),i(b,c))) | $ANS(TARG4).
-P(i(a,a)) | $ANS(TARG4).
% Following negs of 72 singles, all of the known, for BCI.
-P(i(i(p,i(q,r)),i(i(i(s,s),i(t,q)),i(t,i(p,r))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(i(s,i(r,t)),i(q,i(s,t))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(r,r),i(i(i(q,s),s),t)),i(p,t)))) | $ANS(TARG2).
-P(i(i(p,q),i(i(q,i(i(r,r),i(s,t))),i(s,i(p,t))))) | $ANS(TARG2).
-P(i(i(i(i(i(p,p),i(q,r)),r),s),i(i(s,t),i(q,t)))) | $ANS(TARG2).
-P(i(i(i(i(p,q),q),r),i(i(i(s,s),i(r,t)),i(p,t)))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(i(i(i(i(s,s),p),r),t),i(q,t)))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(i(i(s,s),i(r,t)),i(q,i(p,t))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(i(i(q,r),r),s)),i(i(s,t),i(q,t)))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(s,i(i(r,i(s,t)),i(q,t))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(i(i(i(r,s),s),t),i(q,t)))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(i(r,i(s,t)),i(s,i(q,t))))) | $ANS(TARG2).
-P(i(i(p,i(i(q,q),i(r,s))),i(i(s,t),i(r,i(p,t))))) | $ANS(TARG2).
-P(i(i(p,i(i(q,q),r)),i(s,i(i(r,i(s,t)),i(p,t))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(r,r),i(q,i(s,t))),i(s,i(p,t))))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(q,i(i(i(s,s),i(r,t)),i(p,t))))) | $ANS(TARG2).
-P(i(p,i(i(i(q,q),i(r,s)),i(i(s,i(p,t)),i(r,t))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(p,r)),i(i(i(s,s),i(r,t)),i(q,t))))) | $ANS(TARG2).
-P(i(i(p,q),i(r,i(i(i(s,s),i(q,i(r,t))),i(p,t))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(i(i(i(s,t),t),q),i(s,r)))) | $ANS(TARG2).
-P(i(p,i(i(i(q,q),i(r,i(p,s))),i(i(t,r),i(t,s))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,i(r,s))),i(i(t,q),i(r,i(t,s))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(r,r),i(i(i(s,p),q),t)),i(s,t)))) | $ANS(TARG2).
-P(i(i(p,i(i(q,q),i(r,s))),i(i(t,p),i(r,i(t,s))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(i(i(i(s,q),r),t),i(s,t)))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(i(s,i(t,q)),i(t,i(s,r))))) | $ANS(TARG2).
-P(i(i(p,i(i(q,q),r)),i(i(r,i(s,t)),i(s,i(p,t))))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(i(i(s,s),i(t,p)),i(q,i(t,r))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(i(i(q,r),i(i(s,q),r)),t)),i(s,t))) | $ANS(TARG2).

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-P(i(i(i(p,p),i(i(i(q,r),r),s)),i(i(t,q),i(t,s)))) | $ANS(TARG2).
-P(i(i(i(p,p),i(i(i(q,r),s),t)),i(i(r,s),i(q,t)))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,i(r,s))),i(i(s,t),i(r,i(q,t))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,i(r,s))),i(i(t,r),i(t,i(q,s))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,i(r,s))),i(r,i(i(s,t),i(q,t))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,i(r,s))),i(r,i(i(t,q),i(t,s))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,i(r,s))),i(t,i(i(t,r),i(q,s))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(q,i(i(s,i(r,t)),i(s,t))))) | $ANS(TARG2).
-P(i(i(i(p,p),q),i(i(i(i(q,i(r,s)),s),t),i(r,t)))) | $ANS(TARG2).
-P(i(i(p,i(i(q,q),i(r,s))),i(i(t,r),i(t,i(p,s))))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(i(s,i(i(t,t),p)),i(q,i(s,r))))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(q,i(i(i(s,s),i(t,p)),i(t,r))))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(s,i(i(i(t,t),i(s,q)),i(p,r))))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(q,i(i(s,i(i(t,t),p)),i(s,r))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(i(i(r,r),i(s,t)),t),p),i(s,q)))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(i(q,r),r),i(i(s,s),t)),i(p,t)))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(i(r,p),q),i(i(s,s),t)),i(r,t)))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(r,r),i(s,i(q,t))),i(p,i(s,t))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(r,r),i(s,i(t,p))),i(t,i(s,q))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(r,i(i(s,s),i(q,t))),i(p,i(r,t))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(r,i(i(s,s),i(t,p))),i(t,i(r,q))))) | $ANS(TARG2).
-P(i(i(p,q),i(p,i(i(i(r,r),i(s,i(q,t))),i(s,t))))) | $ANS(TARG2).
-P(i(i(p,q),i(r,i(i(i(s,s),i(t,i(r,p))),i(t,q))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(i(r,r),i(p,s))),i(i(s,t),i(q,t))))) | $ANS(TARG2).
-P(i(p,i(i(i(q,q),i(r,i(p,s))),i(i(s,t),i(r,t))))) | $ANS(TARG2).
-P(i(p,i(i(i(q,q),i(r,i(s,t))),i(i(p,s),i(r,t))))) | $ANS(TARG2).
-P(i(p,i(i(i(q,q),i(r,s)),i(i(t,i(p,r)),i(t,s))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(i(r,r),i(p,s))),i(i(t,q),i(t,s))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(i(r,r),i(s,t))),i(i(p,s),i(q,t))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(i(r,r),s)),i(i(s,i(p,t)),i(q,t))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(p,r)),i(i(i(s,s),i(t,q)),i(t,r))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(r,s)),i(i(i(t,t),i(p,r)),i(q,s))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(p,r)),i(i(s,i(i(t,t),q)),i(s,r))))) | $ANS(TARG2).
-P(i(p,i(i(p,q),i(i(i(r,r),i(s,i(q,t))),i(s,t))))) | $ANS(TARG2).
-P(i(p,i(i(q,r),i(i(i(s,s),i(r,i(p,t))),i(q,t))))) | $ANS(TARG2).
-P(i(p,i(i(p,q),i(i(r,i(i(s,s),i(q,t))),i(r,t))))) | $ANS(TARG2).
-P(i(p,i(i(q,r),i(i(r,i(i(s,s),i(p,t))),i(q,t))))) | $ANS(TARG2).
-P(i(p,i(i(q,r),i(i(s,i(i(t,t),i(p,q))),i(s,r))))) | $ANS(TARG2).
-P(i(i(i(i(i(p,p),i(q,r)),r),s),i(i(t,q),i(t,s)))) | $ANS(TARG2).
-P(i(i(i(i(i(p,p),q),r),s),i(i(q,i(t,r)),i(t,s)))) | $ANS(TARG2).
-P(i(i(i(i(p,i(q,r)),r),s),i(i(i(t,t),p),i(q,s)))) | $ANS(TARG2).
-P(i(i(i(i(p,q),q),r),i(i(i(s,s),i(t,p)),i(t,r)))) | $ANS(TARG2).
-P(i(i(i(i(p,q),r),s),i(i(i(t,t),i(q,r)),i(p,s)))) | $ANS(TARG2).
-P(i(a,a)) | $ANS(I).
-P(i(i(a1,i(b,a2)),i(b,i(a1,a2)))) | $ANS(C).  %  C
-P(i(i(a1,b),i(i(a2,a1),i(a2,b)))) | $ANS(B).  % B
end_of_list.

list(demodulators).
%  (P(i(i(i(i(x,x),i(y,z)),i(y,u)),v),i(i(z,u),v))) = junk).
(i(x,junk) = junk).
(i(junk,x) = junk).
(P(junk) = $T).
```

end_of_list.

As is obvious, I had no idea how to prove that candidate 23 was a single axiom, if such was the case, other than seeking the deduction of one of a number of targets. As you see in the given input file, as targets I chose seventy-two known single axioms, each of *B*, *C*, and *I* and their conjunction, and—although the purpose of their inclusion may be far from obvious—the forty-two deduced steps of the Halleck proof for candidate 18. (OTTER, fortunately, encourages its users to seek in each run more than one proof; indeed, I typically assign the value -1 to max_proofs, instructing the program to find many proofs in the run if possible.)

Two forces motivated the inclusion of the cited forty-two targets. First, if and when the program, OTTER, succeeded in proving one or more of the forty-two, I viewed such as a sign of progress; such inclusions are in part for monitoring. Second, and perhaps more important, I conjectured that I might need to iterate, basing the next run on the use of proof steps of proofs of one or more of the forty-two. Both such iteration and use is common to my approach to proof shortening. Yes, I again borrowed from proof shortening in my attempt to find a so-called first proof.

It worked; OTTER succeeded; despite an oversight on my part, all went well. As for the oversight, I failed to include as resonators the correspondents of each of *B*, *C*, and *I*, including but two of them. This failure delayed the program from returning a proof of the join of the three quickly after finding all three proofs; indeed, you had best always include as resonators or hints the members of a conjunction that is a target, each with a small assigned value (to give each a high priority for being chosen to initiate the application of the chosen inference rules).

I offer you two proofs that were completed with use of the cited input file, one of the conjunction (join) and one of an already-known single axiom.

A Proof from Candidate 23 of the Conjunction

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----- Otter 3.3g-work, Jan 2005 -----
The process was started by wos on elephant.mcs.anl.gov,
Tue May 12 10:23:29 2009
The command was "otter".  The process ID is 9983.

-----> EMPTY CLAUSE at 6452.23 sec ----> 278140 [hyper,2,273019,205,274312] $ANS(all).

Length of proof is 52.  Level of proof is 19.

---------- PROOF ----------

1 [] -P(i(x,y))| -P(x)|P(y).
2 [] -P(i(i(a1,i(b,a2)),i(b,i(a1,a2))))| -P(i(a1,a1))| -P(i(i(a1,b),i(i(a2,a1),i(a2,b))))|$ANS(all).
3 [] P(i(i(u,v),i(i(i(w,x),x),i(i(y,y),u)),i(w,v))).
124 [hyper,1,3,3] P(i(i(i(x,y),y),i(i(z,z),i(u,v))),i(x,i(i(i(i(w,v6),v6),i(i(v7,v7),u)),i(w,v)))).
125 [hyper,1,124,124] P(i(i(i(x,x),i(i(y,y),i(z,u))),i(i(i(i(v,w),w),i(i(v6,v6),i(i(i(v7,v8),v8),
  i(i(v9,v9),z)))),i(v,i(v7,u)))).
126 [hyper,1,3,124] P(i(i(i(i(x,y),y),i(i(z,z),i(i(i(u,v),v),i(i(w,w),i(v6,v7))))),i(x,i(u,i(i(i(i(v8,v9),v9),
  i(i(v10,v10),v6)),i(v8,v7))))).
127 [hyper,1,124,3] P(i(x,i(i(i(i(y,z),z),i(i(u,u),i(x,v))),i(y,v)))).
128 [hyper,1,127,127] P(i(i(i(i(x,y),y),i(i(z,z),i(i(u,i(i(i(i(v,w),w),i(i(v6,v6),i(u,v7))),
  i(v,v7))),v8))),i(x,v8))).
129 [hyper,1,124,127] P(i(x,i(i(i(i(y,z),z),z),i(i(u,u),i(i(i(x,v),v),w))),i(y,w)))).
130 [hyper,1,3,127] P(i(i(i(i(x,y),y),i(i(z,z),u)),i(x,i(i(i(i(v,w),w),i(i(v6,v6),i(u,v7))),i(v,v7))))).
131 [hyper,1,127,124] P(i(i(i(i(x,y),y),i(i(z,z),i(i(i(i(i(u,v),v),i(i(w,w),i(v6,v7))),i(u,i(i(i(i(v8,v9),v9),
  i(i(v10,v10),v6)),i(v8,v7)))),v11))),i(x,v11))).
```

132 [hyper,1,127,3] P(i(i(i(i(x,y),y),i(i(z,z),i(i(i(u,v),i(i(i(i(w,v6),v6),i(i(v7,v7),u)),i(w,v))),v8))),i(x,v8))).
133 [hyper,1,125,125] P(i(i(i(i(x,y),y),i(i(z,z),i(i(i(u,v),v),i(i(w,w),i(i(i(v6,v7),v7),i(i(v8,v8),v9)))))),
i(x,i(u,i(v6,v9))))).
134 [hyper,1,124,125] P(i(i(i(x,x),i(y,z)),i(i(i(i(u,v),v),i(i(w,w),i(i(i(v6,v7),v7),i(i(v8,v8),y)))),i(u,i(v6,z))))).
136 [hyper,1,125,3] P(i(i(i(i(x,y),y),i(i(z,z),i(i(i(u,v),v),i(i(w,w),v6)))),i(x,i(u,v6)))).
138 [hyper,1,127,129] P(i(i(i(i(x,y),y),i(i(z,z),i(i(u,i(i(i(i(v,w),w),i(i(v6,v6),i(i(i(u,v7),v7),v8))),
i(v,v8))),v9))),i(x,v9))).
149 [hyper,1,136,127] P(i(x,i(i(x,i(i(y,y),z)),z))).
152 [hyper,1,136,3] P(i(i(x,i(i(y,y),z)),i(x,z))).
158 [hyper,1,126,127] P(i(x,i(i(x,i(i(y,y),i(z,u))),i(i(i(i(v,w),w),i(i(v6,v6),z)),i(v,u))))).
181 [hyper,1,152,149] P(i(i(i(x,x),y),y)).
184 [hyper,1,152,129] P(i(x,i(i(i(i(x,y),y),z),z))).
185 [hyper,1,152,127] P(i(x,i(i(x,y),y))).
205 [hyper,1,181,181] P(i(x,x)).
233 [hyper,1,3,185] P(i(i(i(i(x,y),y),i(i(z,z),u)),i(x,i(i(u,v),v)))).
247 [hyper,1,131,130] P(i(i(i(x,x),i(i(y,y),i(i(i(z,u),u),i(i(v,v),w)))),i(i(i(v6,v6),i(i(v7,v7),i(w,v8))),
i(z,v8)))).
261 [hyper,1,185,184] P(i(i(i(x,i(i(i(i(x,y),y),z),z)),u),u)).
348 [hyper,1,132,134] P(i(i(i(i(x,y),y),z),i(x,z))).
351 [hyper,1,128,134] P(i(i(i(i(i(x,y),y),z),z),u),i(x,u))).
359 [hyper,1,185,348] P(i(i(i(i(i(i(x,y),y),z),i(x,z)),u),u)).
365 [hyper,1,3,348] P(i(i(i(i(x,y),y),i(i(z,z),i(i(i(u,v),v),w))),i(x,i(u,w)))).
367 [hyper,1,348,133] P(i(i(x,i(i(y,y),i(i(i(z,u),u),i(i(v,v),i(i(i(w,v6),v6),i(i(v7,v7),v8)))))),
i(x,i(z,i(w,v8))))).
454 [hyper,1,138,134] P(i(i(i(i(i(i(i(i(x,y),y),z),z),u),u),v),i(x,v))).
467 [hyper,1,134,261] P(i(i(i(i(x,y),y),i(i(z,z),i(i(i(u,v),v),i(i(w,w),v6)))),
i(x,i(u,i(i(i(i(v6,v7),v7),v8),v8))))).
706 [hyper,1,134,359] P(i(i(i(i(x,y),y),i(i(z,z),i(i(i(u,v),v),i(i(w,w),i(i(i(v6,v7),v7),v8))))),
i(x,i(u,i(v6,v8))))).
1163 [hyper,1,233,134] P(i(i(x,y),i(i(i(i(i(i(z,u),u),i(i(v,v),x)),i(z,y)),w),w))).
9084 [hyper,1,454,247] P(i(i(x,i(i(y,y),z)),i(i(i(u,u),i(i(v,v),i(z,w))),i(x,w)))).
12473 [hyper,1,351,365] P(i(x,i(i(x,i(i(y,y),i(i(i(z,u),u),v))),i(z,v)))).
12552 [hyper,1,365,158] P(i(i(x,y),i(i(z,i(i(u,u),x)),i(z,y)))).
14089 [hyper,1,233,12473] P(i(i(i(i(x,y),y),z),i(i(i(x,z),u),u))).
14311 [hyper,1,12552,14089] P(i(i(x,i(i(y,y),i(i(i(z,u),u),v))),i(x,i(i(i(z,v),w),w)))).
150871 [hyper,1,14311,1163] P(i(i(i(x,y),i(i(i(i(z,i(i(u,u),x)),i(z,y)),v),v))).
152983 [hyper,1,14311,150871] P(i(i(i(x,y),i(i(i(z,i(i(z,i(i(u,u),x)),y)),v),v))).
161834 [hyper,1,467,152983] P(i(x,i(i(x,y),i(i(i(i(y,z),z),u),u)))).
161845 [hyper,1,367,152983] P(i(i(x,i(i(i(y,z),z),i(i(u,u),v))),i(x,i(y,v)))).
188740 [hyper,1,161845,161834] P(i(i(x,y),i(x,i(i(y,z),z)))).
188882 [hyper,1,188740,188740] P(i(i(i(x,y),i(i(i(x,i(i(y,z),z)),u),u))).
189719 [hyper,1,185,188882] P(i(i(i(i(i(x,y),i(i(i(x,i(i(y,z),z)),u),u)),v),v),v)).
200414 [hyper,1,706,188882] P(i(i(i(x,i(i(i(y,z),z),u)),i(x,i(y,u)))).
202113 [hyper,1,200414,9084] P(i(i(i(x,i(i(y,y),z)),i(i(i(u,u),i(z,v)),i(x,v)))).
263754 [hyper,1,200414,202113] P(i(i(i(x,i(i(y,y),z)),i(i(z,u),i(x,u)))).
265069 [hyper,1,200414,263754] P(i(i(i(x,i(i(y,y),i(z,u))),i(z,i(x,u)))).
265085 [hyper,1,189719,263754] P(i(i(i(i(x,i(i(y,z),z)),u),i(i(x,y),u))).
273019 [hyper,1,265085,265069] P(i(i(i(x,i(y,z)),i(y,i(x,z)))).
273025 [hyper,1,265085,263754] P(i(i(i(x,y),i(i(y,z),i(x,z)))).
274312 [hyper,1,273019,273025] P(i(i(i(x,y),i(i(z,x),i(z,y)))).

A Proof from Candidate of a Known Single Axiom

----- Otter 3.3g-work, Jan 2005 -----
The process was started by wos on elephant.mcs.anl.gov,
Tue May 12 10:23:29 2009
The command was "otter". The process ID is 9983.

----> UNIT CONFLICT at 6495.88 sec ----> 279581 [binary,279580.1,56.1] $ANS(TARG2).

Length of proof is 51. Level of proof is 20.

---------- PROOF ----------

1 [] -P(i(x,y))| -P(x)|P(y).
3 [] P(i(i(u,v),i(i(i(i(w,x),x),i(i(y,y),u)),i(w,v)))).
56 [] -P(i(i(i(p,p),i(q,r)),i(i(i(i(r,s),s),t),i(q,t))))|$ANS(TARG2).
124 [hyper,1,3,3] P(i(i(i(i(x,y),y),i(i(z,z),i(u,v))),i(x,i(i(i(i(w,v6),v6),i(i(v7,v7),u)),i(w,v))))).
125 [hyper,1,124,124] P(i(i(i(i(x,x),i(i(y,y),i(z,u))),i(i(i(i(v,w),w),i(i(v6,v6),i(i(i(v7,v8),v8),
i(i(v9,v9),z)))),i(v,i(v7,u))))).
126 [hyper,1,3,124] P(i(i(i(i(x,y),y),i(i(z,z),i(i(i(u,v),v),i(i(w,w),i(v6,v7))))),i(x,i(u,i(i(i(i(v8,v9),v9),
i(i(v10,v10),v6)),i(v8,v7)))))).
127 [hyper,1,124,3] P(i(x,i(i(i(i(y,z),z),i(i(u,u),i(x,v))),i(y,v)))).
128 [hyper,1,127,127] P(i(i(i(i(x,y),y),i(i(z,z),i(i(u,i(i(i(i(v,w),w),i(i(v6,v6),i(u,v7))),
i(v,v7))),v8))),i(x,v8))).
129 [hyper,1,124,127] P(i(x,i(i(i(i(y,z),z),i(i(u,u),i(i(i(x,v),v),w))),i(y,w)))).
130 [hyper,1,3,127] P(i(i(i(i(x,y),y),i(i(z,z),u)),i(x,i(i(i(i(v,w),w),i(i(v6,v6),
i(u,v7))),i(v,v7))))).
131 [hyper,1,127,124] P(i(i(i(i(x,y),y),i(i(z,z),i(i(i(i(i(u,v),v),i(i(w,w),i(v6,v7))),i(u,i(i(i(i(v8,v9),v9),
i(i(v10,v10),v6)),i(v8,v7)))),v11))),i(x,v11))).
132 [hyper,1,127,3] P(i(i(i(i(x,y),y),i(i(z,z),i(i(i(u,v),i(i(i(i(w,v6),v6),i(i(v7,v7),u)),
i(w,v))),v8))),i(x,v8))).
133 [hyper,1,125,125] P(i(i(i(i(x,y),y),i(i(z,z),i(i(i(u,v),v),i(i(w,w),i(i(i(v6,v7),v7),
i(i(v8,v8),v9))))),i(x,i(u,i(v6,v9))))).
134 [hyper,1,124,125] P(i(i(i(i(x,x),i(y,z)),i(i(i(i(u,v),v),i(i(w,w),i(i(i(v6,v7),v7),
i(i(v8,v8),y)))),i(u,i(v6,z))))).
136 [hyper,1,125,3] P(i(i(i(i(x,y),y),i(i(z,z),i(i(i(u,v),v),i(i(w,w),v6)))),i(x,i(u,v6)))).
138 [hyper,1,127,129] P(i(i(i(i(x,y),y),i(i(z,z),i(i(u,i(i(i(i(v,w),w),i(i(v6,v6),
i(i(i(u,v7),v7),v8))),i(v,v8))),v9))),i(x,v9))).
149 [hyper,1,136,127] P(i(x,i(i(x,i(i(y,y),z)),z))).
152 [hyper,1,136,3] P(i(i(x,i(i(y,y),z)),i(x,z))).
158 [hyper,1,126,127] P(i(x,i(i(x,i(i(y,y),i(z,u))),i(i(i(i(v,w),w),i(i(v6,v6),z)),i(v,u))))).
164 [hyper,1,129,149] P(i(i(i(i(x,y),y),i(i(z,z),i(i(i(i(u,i(i(u,i(i(v,v),w)),w)),v6),v6),v7))),i(x,v7))).
184 [hyper,1,152,129] P(i(x,i(i(i(i(x,y),y),z),z))).
185 [hyper,1,152,127] P(i(x,i(i(x,y),y))).
233 [hyper,1,3,185] P(i(i(i(i(x,y),y),i(i(z,z),u)),i(x,i(i(u,v),v)))).
247 [hyper,1,131,130] P(i(i(i(x,x),i(i(y,y),i(i(i(z,u),u),i(i(v,v),w)))),i(i(i(v6,v6),i(i(v7,v7),
i(w,v8))),i(z,v8)))).
261 [hyper,1,185,184] P(i(i(i(x,i(i(i(i(x,y),y),z),z)),u),u)).
348 [hyper,1,132,134] P(i(i(i(i(x,y),y),z),i(x,z))).
351 [hyper,1,128,134] P(i(i(i(i(i(i(x,y),y),z),z),u),i(x,u))).
359 [hyper,1,185,348] P(i(i(i(i(i(i(x,y),y),z),i(x,z)),u),u)).
365 [hyper,1,3,348] P(i(i(i(i(x,y),y),i(i(z,z),i(i(i(u,v),v),w))),i(x,i(u,w)))).
367 [hyper,1,348,133] P(i(i(x,i(i(y,y),i(i(i(z,u),u),i(i(v,v),i(i(i(w,v6),v6),i(i(v7,v7),v8)))))),i(x,i(z,i(w,v8))))).
454 [hyper,1,138,134] P(i(i(i(i(i(i(i(i(x,y),y),z),z),u),u),v),i(x,v))).
467 [hyper,1,134,261] P(i(i(i(i(x,y),y),i(i(z,z),i(i(i(u,v),v),i(i(w,w),v6)))),

i(x,i(u,i(i(i(i(v6,v7),v7),v8),v8))))).
706 [hyper,1,134,359] P(i(i(i(i(x,y),y),i(i(z,z),i(i(i(u,v),v),i(i(w,w),i(i(i(v6,v7),v7),v8))))),
i(x,i(u,i(v6,v8))))).
1163 [hyper,1,233,134] P(i(i(x,y),i(i(i(i(i(i(z,u),u),i(i(v,v),x)),i(z,y)),w),w))).
1813 [hyper,1,164,158] P(i(i(x,y),i(i(i(z,z),x),y))).
9084 [hyper,1,454,247] P(i(i(x,i(i(y,y),z)),i(i(i(u,u),i(i(v,v),i(z,w))),i(x,w)))).
12473 [hyper,1,351,365] P(i(x,i(i(x,i(i(y,y),i(i(i(z,u),u),v))),i(z,v)))).
12552 [hyper,1,365,158] P(i(i(x,y),i(i(z,i(i(u,u),x)),i(z,y)))).
14089 [hyper,1,233,12473] P(i(i(i(i(x,y),y),z),i(i(i(x,z),u),u))).
14311 [hyper,1,12552,14089] P(i(i(x,i(i(y,y),i(i(i(z,u),u),v))),i(x,i(i(i(z,v),w),w)))).
150871 [hyper,1,14311,1163] P(i(i(x,y),i(i(i(i(z,i(i(u,u),x)),i(z,y)),v),v))).
152983 [hyper,1,14311,150871] P(i(i(x,y),i(i(i(z,i(i(z,i(i(u,u),x)),y)),v),v))).
161834 [hyper,1,467,152983] P(i(x,i(i(x,y),i(i(i(i(y,z),z),u),u)))).
161845 [hyper,1,367,152983] P(i(i(x,i(i(i(y,z),z),i(i(u,u),v))),i(x,i(y,v)))).
188740 [hyper,1,161845,161834] P(i(i(x,y),i(x,i(i(y,z),z)))).
188882 [hyper,1,188740,188740] P(i(i(x,y),i(i(i(x,i(i(y,z),z)),u),u))).
189719 [hyper,1,185,188882] P(i(i(i(i(x,y),i(i(i(x,i(i(y,z),z)),u),u)),v),v)).
200414 [hyper,1,706,188882] P(i(i(x,i(i(i(y,z),z),u)),i(x,i(y,u)))).
202113 [hyper,1,200414,9084] P(i(i(x,i(i(y,y),z)),i(i(i(u,u),i(z,v)),i(x,v)))).
263754 [hyper,1,200414,202113] P(i(i(x,i(i(y,y),z)),i(i(z,u),i(x,u)))).
265085 [hyper,1,189719,263754] P(i(i(i(x,i(i(y,z),z)),u),i(i(x,y),u))).
273025 [hyper,1,265085,263754] P(i(i(x,y),i(i(y,z),i(x,z)))).
274317 [hyper,1,265085,273025] P(i(i(x,y),i(i(i(i(y,z),z),u),i(x,u)))).
279580 [hyper,1,1813,274317] P(i(i(i(x,x),i(y,z)),i(i(i(i(z,u),u),v),i(y,v)))).

Summarizing, I had answered an open question by showing that candidate 23 is in fact a (shortest) single axiom for the *BCI* logic. The approach I used, based on how I would have attacked the study if the goal were a shorter proof, had yielded a contribution to logic. But, in an attempt to adhere strictly to science, I offer you a mystery. An analysis of the 52-step proof given earlier showed that virtually none of the set of forty-two resonators as formulas or the set of twenty-three resonators occurs among the fifty-two. In fact, even as resonators, treating all variables as indistinguishable, virtually none of either set is present among the fifty-two if also viewed as resonators. So, why was success the result? I leave that to some scholar or to somebody with much curiosity. I do, however, note that the presence of the given resonators markedly reorders the formulas that are chosen to drive the reasoning. Evidently that reordering enables the program to get to what is needed to complete the sought-after proof, no trivial feat in that the question had remained open despite being attacked by fine minds. Indeed, many formulas that would have been delayed for consideration are moved up a great deal because of matching a resonator and, therefore, given a priority for use much greater than that which would have been assigned based purely on symbol count. The consequences of this sharp reordering are difficult to analyze.

With the answer to the at-one-time open question concerning candidate 23 in hand, I felt optimistic about answering other questions about possible single axioms for the *BCI* logic, questions posed by Ulrich. History has lost the reason, if there was one, but next in order was my study of candidate 51, the following.

P(i(i(i(i(u,v),i(i(w,u),v)),i(i(x,x),y)),i(w,y))). % BCI-Candidate 51

My approach was the simplest possible: I merely modified the given input file by replacing (in the sos list) candidate 23 by candidate 51. I could have, and perhaps should have, added as a target the newest single axioms, 18, 10, and 23, each of which was found during the study that is the basis of this notebook; but, I did not. If you had submitted the run under discussion, you might have lost patience, as you can see from the proof I give now; indeed, much CPU time was required to get anywhere.

A Proof Establishing Candidate 51 to Be a Shortest Single Axiom

----- Otter 3.3g-work, Jan 2005 -----

```
The process was started by wos on elephant.mcs.anl.gov,
Wed Apr  8 18:47:40 2009
The command was "otter".  The process ID is 8480.

----> UNIT CONFLICT at 103383.63 sec ----> 1346204 [binary,1346203.1,74.1] $ANS(TARG2).

Length of proof is 14.  Level of proof is 11.

---------- PROOF ----------

1 [] -P(i(x,y))| -P(x)|P(y).
3 [] P(i(i(i(i(u,v),i(i(w,u),v)),i(i(x,x),y)),i(w,y))).
74 [] -P(i(i(i(p,p),i(i(i(q,r),i(i(s,q),r)),t)),i(s,t)))|$ANS(TARG2).
153 [hyper,1,3,3] P(i(i(x,y),i(i(i(z,z),x),y))).
155 [hyper,1,3,153] P(i(x,i(i(x,y),y))).
159 [hyper,1,3,155] P(i(x,i(i(y,z),i(i(x,y),z)))).
161 [hyper,1,155,3] P(i(i(i(i(i(i(x,y),i(i(z,x),y)),i(i(u,u),v)),i(z,v)),w),w)).
193 [hyper,1,159,3] P(i(i(x,y),i(i(i(i(i(i(z,u),i(i(v,z),u)),i(i(w,w),v6)),i(v,v6)),x),y))).
1264 [hyper,1,161,193] P(i(i(i(i(i(i(x,y),i(i(z,x),y)),i(i(u,u),v)),i(z,v)),i(i(i(w,v6),
  i(i(v7,w),v6)),i(i(v8,v8),v9))),i(v7,v9))).
1267 [hyper,1,3,193] P(i(x,i(i(x,i(i(i(y,z),i(i(u,y),z)),i(i(v,v),w))),i(u,w)))).
116909 [hyper,1,3,1267] P(i(i(x,y),i(z,i(i(z,x),y)))).
117401 [hyper,1,116909,116909] P(i(x,i(i(x,i(y,z)),i(u,i(i(u,y),z))))).
117562 [hyper,1,155,116909] P(i(i(i(i(x,y),i(z,i(i(z,x),y))),u),u)).
119007 [hyper,1,3,117401] P(i(x,i(y,i(i(y,i(z,u)),i(i(x,z),u))))).
120049 [hyper,1,117562,119007] P(i(x,i(i(x,i(y,z)),i(i(i(i(u,v),i(w,i(i(w,u),v))),y),z)))).
1343562 [hyper,1,1264,120049] P(i(i(i(i(x,y),i(i(z,x),y)),u),i(z,u))).
1346203 [hyper,1,153,1343562] P(i(i(i(x,x),i(i(i(y,z),i(i(u,y),z)),v)),i(u,v))).
```

Did you enjoy witnessing OTTER's power, retaining more than 1,300,000 new conclusions before the desired proof was completed, and using more than 103,000 CPU-seconds? These two numbers suggest how difficult was the open question that was solved.

From Ulrich's viewpoint, the study was complete. In particular, he typically (in such studies) seeks the proof of some known single axiom. Halleck, on the other hand, appears to prefer to call a study of this type complete when and if a proof is found that deduces, from the candidate, the conjunction (join) of *B*, *C*, and *I*. Therefore, if memory serves, I considered his goal merited seeking, and I had OTTER seek a proof of the join from the sole hypothesis Candidate 51. But, since the modified input file had not yielded what I sought, how was I to proceed? For a related question, how could I borrow from my various methodologies for proof shortening?

Iteration is common in my attempts to find a shorter proof than that in hand. I often take the results of one run and use them in some way for a next run, whether progress is occurring or not. Therefore, I (as is typical of my proof-shortening approaches) took all the proof steps that occur in the output that yielded the 14-step proof just given and sorted them to remove duplicates. now, you might immediately wonder whether I took proof steps from proofs of what might be termed intermediate targets, those corresponding to items in the Halleck 42-step proof. Indeed I did. As remarked earlier, those targets were included with the intention of, if necessary, using such intermediate proofs in an iterative attack. After sorting, I had twenty-six formulas.

I then amended the modified input file by placing, just after weight_list(pick_and_purge, the following.

```
% Following 26 sorted proof steps from cand51, temp.halleck.bci.cand51.out1b.
weight(P(i(i(i(i(i(i(x,y),i(i(z,x),y)),i(i(u,u),v)),i(z,v)),i(i(i(w,v6),i(i(v7,w),v6)),i(i(v8,v8),v9))),
  i(v7,v9))),-5).
```

```
weight(P(i(i(i(i(i(i(x,y),i(i(z,x),y)),i(i(u,u),v)),i(z,v)),w),w)),-5).
weight(P(i(i(i(i(x,y),i(i(i(z,z),x),y)),i(u,v)),i(i(i(i(i(i(w,v6),i(i(v7,w),v6)),i(i(v8,v8),v9)),
  i(v7,v9)),u),v))),-5).
weight(P(i(i(i(i(x,y),i(i(z,x),y)),u),i(z,u))),-5).
weight(P(i(i(i(i(x,y),i(z,i(i(z,x),y))),u),u)),-5).
weight(P(i(i(i(i(x,y),y),i(i(z,z),u)),i(x,u))),-5).
weight(P(i(i(i(x,i(i(y,z),i(i(x,y),z))),i(i(i(u,v),i(i(w,u),v)),i(i(v6,v6),v7))),i(w,v7))),-5).
weight(P(i(i(i(x,x),i(i(i(y,z),i(i(u,y),z)),v)),i(u,v))),-5).
weight(P(i(i(i(x,x),i(y,z)),i(i(i(u,u),y),z))),-5).
weight(P(i(i(i(x,x),y),y)),-5).
weight(P(i(i(x,i(i(y,y),z)),i(x,z))),-5).
weight(P(i(i(x,y),i(i(i(i(i(z,u),i(i(v,z),u)),i(i(w,w),v6)),i(v,v6)),x),y))),-5).
weight(P(i(i(x,y),i(i(i(i(z,u),i(i(i(v,v),z),u)),x),y))),-5).
weight(P(i(i(x,y),i(i(i(z,i(i(u,v),i(i(z,u),v))),x),y))),-5).
weight(P(i(i(x,y),i(i(i(z,i(i(z,x),y)),u),u))),-5).
weight(P(i(i(x,y),i(i(i(z,z),x),y))),-5).
weight(P(i(i(x,y),i(z,i(i(z,x),y)))),-5).
weight(P(i(x,i(i(x,i(i(i(y,z),i(i(u,y),z)),i(i(v,v),w))),i(u,w)))),-5).
weight(P(i(x,i(i(x,i(i(i(i(y,z),i(u,i(i(u,y),z))),v)),v))),-5).
weight(P(i(x,i(i(x,i(i(i(i(y,z),z),i(i(u,u),v))),i(y,v)))),-5).
weight(P(i(x,i(i(x,i(y,z)),i(i(i(i(u,v),i(w,i(i(w,u),v))),y),z)))),-5).
weight(P(i(x,i(i(x,i(y,z)),i(u,i(i(u,y),z))))),-5).
weight(P(i(x,i(i(x,y),y))),-5).
weight(P(i(x,i(i(y,z),i(i(x,y),z)))),-5).
weight(P(i(x,i(y,i(i(y,i(z,u)),i(i(x,z),u))))),-5).
weight(P(i(x,x)),-5).
```

The cited addition tells the program to prefer, for inference-rule initiation, any item that matches one of the twenty-six weight templates, where (again) all variables are treated as indistinguishable from each other. In other words, I adjoined twenty-six new resonators, one for each of the sorted proof steps from the run that produced the cited 14-step proof. The hope was that, with the additional guidance of its reasoning, OTTER would find its way to a proof of the target conjunction. Again, McCune's program rewarded my effort, with the following proof.

A Proof from Candidate 51 of the BCI Conjunction

```
----- Otter 3.3g-work, Jan 2005 -----
The process was started by wos on octopus.mcs.anl.gov,
Wed May 13 14:24:22 2009
The command was "otter".  The process ID is 23243.

-----> EMPTY CLAUSE at 2912.57 sec ----> 292452 [hyper,2,285131,159,289303] $ANS(all).

Length of proof is 41.  Level of proof is 16.

---------- PROOF ----------

1 [] -P(i(x,y))| -P(x)|P(y).
2 [] -P(i(i(a1,i(b,a2)),i(b,i(a1,a2))))| -P(i(a1,a1))| -P(i(i(a1,b),i(i(a2,a1),i(a2,b))))|$ANS(all).
3 [] P(i(i(i(i(u,v),i(i(w,u),v)),i(i(x,x),y)),i(w,y))).
153 [hyper,1,3,3] P(i(i(x,y),i(i(i(z,z),x),y))).
154 [hyper,1,153,153] P(i(i(i(x,x),i(y,z)),i(i(i(u,u),y),z))).
155 [hyper,1,3,153] P(i(x,i(i(x,y),y))).
```

159 [hyper,1,3,154] P(i(x,x)).
166 [hyper,1,3,155] P(i(x,i(i(y,z),i(i(x,y),z)))).
169 [hyper,1,155,3] P(i(i(i(i(i(i(x,y),i(i(z,x),y)),i(i(u,u),v)),i(z,v)),w),w)).
172 [hyper,1,166,166] P(i(i(x,y),i(i(i(z,i(i(u,v),i(i(z,u),v))),x),y))).
176 [hyper,1,166,155] P(i(i(x,y),i(i(i(z,i(i(z,u),u)),x),y))).
178 [hyper,1,166,153] P(i(i(x,y),i(i(i(i(z,u),i(i(i(v,v),z),u)),x),y))).
179 [hyper,1,166,3] P(i(i(x,y),i(i(i(i(i(i(z,u),i(i(v,z),u)),i(i(w,w),v6)),i(v,v6)),x),y))).
192 [hyper,1,169,172] P(i(i(i(x,i(i(y,z),i(i(x,y),z))),i(i(i(u,v),i(i(w,u),v)),i(i(v6,v6),v7))),i(w,v7))).
213 [hyper,1,3,176] P(i(x,i(i(x,y),i(i(y,z),z)))).
249 [hyper,1,178,179] P(i(i(i(i(x,y),i(i(i(z,z),x),y)),i(u,v)),i(i(i(i(i(i(w,v6),i(i(v7,w),v6)),
i(i(v8,v8),v9)),i(v7,v9)),u),v))).
252 [hyper,1,169,179] P(i(i(i(i(i(i(x,y),i(i(z,x),y)),i(i(u,u),v)),i(z,v)),i(i(i(w,v6),i(i(v7,w),v6)),
i(i(v8,v8),v9))),i(v7,v9))).
256 [hyper,1,3,179] P(i(x,i(i(x,i(i(i(y,z),i(i(u,y),z)),i(i(v,v),w))),i(u,w)))).
299 [hyper,1,166,213] P(i(i(x,y),i(i(i(z,i(i(z,u),i(i(u,v),v))),x),y))).
321 [hyper,1,192,249] P(i(i(i(i(x,y),y),i(i(z,z),u)),i(x,u))).
337 [hyper,1,192,252] P(i(i(x,y),i(z,i(i(z,x),y)))).
353 [hyper,1,256,256] P(i(i(i(x,i(i(x,i(i(i(y,z),i(i(u,y),z)),i(i(v,v),w))),i(u,w))),i(i(i(v6,v7),
i(i(v8,v6),v7)),i(i(v9,v9),v10))),i(v8,v10))).
363 [hyper,1,256,213] P(i(i(i(x,i(i(x,y),i(i(y,z),z))),i(i(i(u,v),i(i(w,u),v)),i(i(v6,v6),v7))),i(w,v7))).
377 [hyper,1,256,299] P(i(i(i(i(x,y),i(i(i(z,i(i(z,u),i(i(u,v),v))),x),y)),i(i(i(w,v6),i(i(v7,w),v6)),
i(i(v8,v8),v9))),i(v7,v9))).
427 [hyper,1,337,337] P(i(x,i(i(x,i(y,z)),i(u,i(i(u,y),z))))).
438 [hyper,1,155,337] P(i(i(i(i(x,y),i(z,i(i(z,x),y))),u),u)).
441 [hyper,1,337,321] P(i(x,i(i(x,i(i(i(y,z),z),i(i(u,u),v))),i(y,v)))).
500 [hyper,1,3,427] P(i(x,i(y,i(i(y,i(z,u)),i(i(x,z),u))))).
552 [hyper,1,427,441] P(i(i(i(x,i(i(x,i(i(i(y,z),z),i(i(u,u),v))),i(y,v))),i(w,v6)),i(v7,i(i(v7,w),v6)))).
634 [hyper,1,438,500] P(i(x,i(i(x,i(y,z)),i(i(i(u,v),i(w,i(i(w,u),v))),y),z)))).
808 [hyper,1,252,634] P(i(i(i(i(x,y),i(i(z,x),y)),u),i(z,u))).
963 [hyper,1,337,808] P(i(x,i(i(x,i(i(i(y,z),i(i(u,y),z)),v)),i(u,v)))).
976 [hyper,1,155,808] P(i(i(i(i(i(i(x,y),i(i(z,x),y)),u),i(z,u)),v),v)).
985 [hyper,1,808,363] P(i(i(x,y),i(z,i(i(i(i(z,x),y),u),u)))).
987 [hyper,1,808,252] P(i(i(i(i(i(x,x),y),z),i(i(u,u),v)),i(i(y,z),v))).
22961 [hyper,1,987,963] P(i(i(i(i(x,y),y),z),i(x,z))).
22993 [hyper,1,985,22961] P(i(x,i(i(i(i(x,i(i(i(y,z),z),u)),i(y,u)),v),v))).
69867 [hyper,1,808,377] P(i(i(i(i(x,y),i(i(y,z),z)),u),i(x,u))).
280883 [hyper,1,353,552] P(i(i(x,i(i(y,y),z)),i(i(u,x),i(u,z)))).
281445 [hyper,1,69867,280883] P(i(x,i(i(y,i(x,z)),i(y,z)))).
282301 [hyper,1,281445,281445] P(i(i(x,i(i(y,i(i(z,i(y,u)),i(z,u))),v)),i(x,v))).
285131 [hyper,1,282301,281445] P(i(i(x,i(y,z)),i(y,i(x,z)))).
286043 [hyper,1,285131,22993] P(i(i(i(i(x,i(i(i(y,z),z),u)),i(y,u)),v),i(x,v))).
289303 [hyper,1,976,286043] P(i(i(x,y),i(i(z,x),i(z,y)))).

And now I offer a strange (to me) and involved story, a story that eventually brings other researchers into the game. To win the game, or at least try to win, even more of the methodologies I use for proof shortening would come into play. Flushed with success and aware of other open questions (offered by Ulrich) concerning possible shortest single axioms for *BCI*, I chose another to study, namely, the following, candidate 46.

P(i(i(i(i(u,u),i(v,w)),i(i(x,v),w)),y),i(x,y))). % BCI-Candidate 46

I cannot say why I chose that formula to study in the attempt to prove it a single axiom. Innocently—although I, as history shows, had no idea how innocent I was being—I simply took the input file and, emulating what had worked already with candidate 51, replaced candidate 23 with candidate 46. When I began, on April 8, 2009, what turned out to be a long journey, by submitting the corresponding file to OTTER, I

had no idea that I was entering a quagmire.

The experiment immediately deduced *I* and quickly deduced three additional items from Halleck's 42-step proof, all within less than 1 CPU-second. And then, after retaining more than 1,000,000 new conclusions and using more than twenty CPU-hours, another item was proved from among the 42. In a way, patience was rewarded. The proof of the cited item is twenty-eight steps in length. True, the item was neither *B* nor *C*, but its proof could be put to good use, as I would have if proof shortening were the goal. Indeed, I immediately set up the next experiment by adding to the input file resonators, weight templates to direct the reasoning, corresponding to the twenty-eight steps of that last proof. As a reminder, these added resonators do not in any way play the role of lemmas or additional facts; rather, their functional pattern is the key. I was reasonably optimistic about making more progress—and I was mistaken. Although far less time was required to deduce the same items, nothing new was proved.

The action to take was obvious: Behave as if proof shortening were the object. I therefore switched to a breadth-first (level-saturation) search and placed, in addition to what I was using, the twenty-eight proof steps in list(sos). I pause here to discuss the cited action, an action that is reminiscent of the *cramming strategy* that I introduced many years ago when collaborating with other researchers in the study of proof shortening. The idea is to cram or force the added items into use, to do (if possible) double duty, triple duty, or more, that is, to be used throughout a sought-after proof with the side effect of reducing the number of additional deduced steps required to complete a proof. In this case, the program is given a head start, forced to focus on those steps that have been added to the initial set of support before focusing on any newly deduced item.

Again, I thought OTTER would find more proofs, even, perhaps one or both of *B* and *C*. Seven CPU-hours later, the program proved another of the members of the 42-step proof—and that is all. Yes, I was disappointed, and I began to realize I was at least in a marsh and, just perhaps, in a quagmire. So, typical of proof shortening, in the next experiment I increased the value assigned to the max_weight from 48 to 56, allowing more complex information to be retained, and decreased the assigned value of max_distinct_vars from 14 to 12. The latter action was taken to possibly permit the program to explore deeper into the search space. A tiny amount of progress occurred: The program deduced yet one more member of the Halleck 42-step proof, offering a proof of length six. And, as is predictable, I adjoined to list(sos) the six steps and conducted the next experiment, one that yielded nothing.

Now it was April 16, and, from what i had in hand, *B* and *C* were no closer in the context of deducibility. In fact, perhaps candidate 46 was not a single axiom for the *BCI* logic. The story becomes more complicated, more involved, stranger. Specifically, I decided to offer to the research world the candidate-46-problem for possible study, offered it in a column (the President's Column) I regularly write for the AAR newsletter. In that column, I also offered for study the following formula, candidate 42.

```
% P(i(u,i(i(v,w),i(i(i(x,x),i(y,i(u,v))),i(y,w))))). % BCI-Candidate 42,
% unlikely because of i(x,x) variants
```

I noted in that column that candidate 42 was not promising in that it appeared that all formulas deducible from it contain an alphabetic variant of *i*(*ix*,*x*). I did comment, if memory serves, that 46 was more promising.

As the column was being offered on the Web, I continued experimenting, using the following input file.

An Input File That Focuses Further on Candidate 46

```
set(hyper_res).
assign(max_weight,48).
assign(change_limit_after,800).
assign(new_max_weight,33).
assign(max_proofs,-1).
clear(print_kept).
```

```
%  set(ancestor_subsume).
set(back_sub).
% clear(for_sub).
clear(print_back_sub).
clear(print_kept).
clear(print_new_demod).
clear(print_back_demod).
clear(print_back_sub).
assign(max_distinct_vars,8).
%  assign(pick_given_ratio,2).
assign(max_mem,750000).
%  assign(max_seconds,2).
assign(report,5400).
set(order_history).
set(input_sos_first).
set(sos_queue).
assign(bsub_hint_wt,1).
set(keep_hint_subsumers).

weight_list(pick_and_purge).
%  Following 32 of 33 prove cand22 a single, deriving the join of BCI, from Halleck 04-13-09.
weight(P(i(i(x,y),i(i(i(i(z,x),y),i(i(u,u),v)),i(z,v)))),-6).  % BCI-candidate-22
weight(P(i(i(i(i(v41,i(v42,v43)),i(i(i(i(v44,v42),v43),i(i(v45,v45),v46)),i(v44,v46))),
  i(i(v47,v47),v4748)),i(v41,v4748))),-6).
weight(P(i(i(i(i(v41,i(i(i(v42,i(v43,v44)),i(i(i(i(v45,v43),v44),i(i(v46,v46),v47)),i(v45,v47))),
  i(i(v4748,v4748),x))),i(v42,x)),i(i(y,y),z)),i(v41,z))),-6).
weight(P(i(i(i(v41,v41),i(i(v42,v42),i(v43,v44))),i(v43,v44))),-6).
weight(P(i(v41,v41)),-6).
weight(P(i(v41,i(i(v41,i(v42,v43)),i(i(i(i(v44,v42),v43),i(i(v45,v45),v46)),i(v44,v46))))),-6).
weight(P(i(v41,i(i(i(i(v42,i(v41,i(v43,v44))),i(i(i(i(v45,v43),v44),i(i(v46,v46),v47)),i(v45,v47))),
  i(i(v4748,v4748),x)),i(v42,x)))),-6).
weight(P(i(i(i(i(v41,i(i(i(v42,v43),i(i(i(i(v44,v42),v43),i(i(v45,v45),v46)),i(v44,v46))),i(v47,v4748))),
  i(i(i(i(x,v47),v4748),i(i(y,y),z)),i(x,z))),i(i(v4749,v4749),v4750)),i(v41,v4750))),-6).
weight(P(i(i(i(i(i(i(v41,i(v42,v43)),i(i(i(i(v44,v42),v43),i(i(v45,v45),v46)),i(v44,v46))),
  i(i(v47,v47),v4748)),i(v41,v4748)),i(x,y)),i(i(i(i(z,x),y),i(i(v4749,v4749),v4750)),i(z,v4750)))),-6).
weight(P(i(i(i(i(i(i(v41,i(v42,v43)),i(i(i(i(v44,v42),v43),i(i(v45,v45),v46)),i(v44,v46))),i(v47,v4748)),
  i(i(i(i(x,v47),v4748),i(i(y,y),z)),i(x,z))),i(i(v4749,v4749),i(i(v4750,v4750),v4751))),i(v41,v4751))),-6).
weight(P(i(i(v41,v42),i(v43,i(i(v43,v41),v42)))),-6).
weight(P(i(i(i(v41,v41),i(v42,v43)),i(i(i(i(v44,v42),v43),i(i(v45,v45),v46)),i(v44,v46)))),-6).
weight(P(i(i(i(i(v41,i(i(i(v42,v43),v43),i(i(v44,v44),v45))),i(v42,v45)),i(i(v46,v46),v47)),i(v41,v47))),-6).
weight(P(i(v41,i(i(v41,i(i(i(v42,v43),v43),i(i(v44,v44),v45))),i(v42,v45)))),-6).
weight(P(i(i(i(i(v41,v42),i(i(v42,i(i(i(v43,v44),v44),i(i(v45,v45),v46))),i(v43,v46))),
  i(i(v47,v47),v4748)),i(v41,v4748))),-6).
weight(P(i(i(v41,i(i(i(v42,v43),v43),i(i(v44,v44),v45))),i(v41,i(v42,v45)))),-6).
weight(P(i(i(i(i(v41,v42),v42),v43),i(v41,v43))),-6).
weight(P(i(v41,i(i(v41,i(i(i(i(v42,v43),v43),v44)),i(v42,v44)))),-6).
weight(P(i(v41,i(i(v41,i(i(i(i(v42,i(v43,v44)),i(i(i(i(v45,v43),v44),i(i(v46,v46),v47)),i(v45,v47))),
  i(i(v4748,v4748),x))),i(v42,x)))),-6).
weight(P(i(i(i(v41,i(i(i(i(v42,i(v41,i(v43,v44))),i(i(i(i(v45,v43),v44),i(i(v46,v46),v47)),i(v45,v47))),
  (i(v4748,v4748),x)),i(v42,x))),i(i(i(y,i(z,v4749)),i(i(i(i(v4750,z),v4749),
  i(i(v4751,v4751),v4752)),i(v4750,v4752))),i(i(v4753,v4753),v4754))),i(y,v4754))),-6).
weight(P(i(i(i(i(v41,i(i(v42,i(i(v42,i(i(i(v43,i(v44,v45)),i(i(i(i(v46,v44),v45),i(i(v47,v47),v4748)),
  i(v46,v4748))),i(i(x,x),y))),i(v43,y))),i(z,v4749))),i(i(i(i(v4750,z),v4749),
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  i(i(v4751,v4751),v4752)),i(v4750,v4752))),i(i(v4753,v4753),v4754)),i(v41,v4754))),-6).
weight(P(i(i(i(i(v41,v42),v43),i(i(v44,v44),v45)),i(i(v42,v43),i(v41,v45)))),-6).
weight(P(i(i(v41,v42),i(i(v43,v41),i(v43,v42)))),-6).
weight(P(i(i(i(v41,i(i(v41,i(v42,v43)),i(i(i(i(v44,v42),v43),i(i(v45,v45),v46)),i(v44,v46)))),
  i(v47,v4748)),i(i(i(i(x,v47),v4748),i(i(y,y),z)),i(x,z)))),-6).
weight(P(i(i(i(i(v41,i(i(i(i(v42,i(i(v42,i(v43,v44)),i(i(i(i(v45,v43),v44),i(i(v46,v46),v47)),i(v45,v47)))),
  i(v4748,x)),i(i(i(i(y,v4748),x),i(i(z,z),v4749)),i(y,v4749))),i(v4750,v4751))),i(i(i(i(v4752,v4750),v4751),
  i(i(v4753,v4753),v4754)),i(v4752,v4754))),i(i(v4755,v4755),v4756)),i(v41,v4756))),-6).
weight(P(i(i(i(v41,i(i(v41,i(i(i(i(v42,i(v43,v44)),i(i(i(i(v45,v43),v44),i(i(v46,v46),v47)),i(v45,v47))),
  i(i(v4748,v4748),x))),i(v42,x))),i(i(i(y,i(z,v4749)),i(i(i(i(v4750,z),v4749),i(i(v4751,v4751),v4752)),
  i(v4750,v4752))),i(i(v4753,v4753),v4754))),i(y,v4754))),-6).
weight(P(i(i(v41,i(v42,v43)),i(i(v44,v41),i(v44,i(v42,v43))))),-6).
weight(P(i(i(i(i(i(i(i(v41,i(v42,v43)),i(i(i(i(v44,v42),v43),i(i(v45,v45),v46)),i(v44,v46))),
  i(i(v47,v47),v4748)),i(v41,v4748)),i(x,y)),i(i(i(i(z,x),y),i(i(v4749,v4749),v4750)),i(z,v4750))),
  i(i(i(v4751,i(v4752,v4753)),i(i(i(i(v4754,v4752),v4753),i(i(v4755,v4755),v4756)),i(v4754,v4756))),
  i(i(x1,x1),y1))),i(v4751,y1))),-6).
weight(P(i(i(i(i(v41,v42),v42),i(v43,v44)),i(v41,i(v43,v44)))),-6).
weight(P(i(v41,i(i(v42,i(v41,i(v43,v44))),i(v42,i(v43,v44))))),-6).
weight(P(i(v41,i(i(v42,i(v41,v43)),i(v42,v43)))),-6).
weight(P(i(i(v41,i(i(v42,i(i(v43,i(v42,v44)),i(v43,v44))),v45)),i(v41,v45))),-6).
weight(P(i(i(v41,i(v42,v43)),i(v42,i(v41,v43)))),-6).
% Following 42 apparently prove candidate 18 a new single axiom, from Halleck.
weight(P(i(i(x,y),i(i(i(i(z,z),i(y,u)),u),x2),i(x,x2)))),-4). % BCI candidate 18
weight(P(i(i(i(i(i(x,x),i(i(i(i(i(i(y,y),i(z,u)),u),x1),i(u,x1)),v)),v),w),i(i(u,z),w))),-4).
weight(P(i(i(x,i(y,z)),i(i(u,y),i(u,i(x,z))))),-4).
weight(P(i(i(i(i(i(x,x),i(i(i(y,z),u),x2)),x2),u),i(i(i(i(i(v,v),i(i(i(i(i(w,w),
  i(z,x)),x),y),i(y,y)),z)),z),u),u))),-4).
weight(P(i(i(i(i(i(x,x),i(i(i(i(i(i(y,y),i(z,u)),u),x2),i(u,x2)),v)),v),w),i(i(x,y),
  i(x,i(i(i(z,z),i(i(i(u,z),w),i(y,x1))),x1))))),-4).
weight(P(i(i(x,y),i(x,i(i(i(z,z),i(i(i(u,i(x2,u)),i(i(v,x2),i(v,i(u,u)))),i(y,w))),w)))),-4).
weight(P(i(i(x,i(i(i(i(y,y),i(z,u)),u),x2)),i(x,i(i(u,z),i(u,x2))))),-4).
weight(P(i(i(x,i(y,z)),i(i(u,x2),i(u,i(i(i(u,u),i(x2,y)),i(x,z)))))),-4).
weight(P(i(i(x,y),i(x,i(i(i(z,z),i(y,i(i(i(i(u,u),i(x2,u)),u),v))),i(i(w,x2),i(w,v)))))),-4).
weight(P(i(i(x,i(y,z)),i(x,i(i(u,i(z,x2)),i(y,i(u,x2)))))),-4).
weight(P(i(i(x,i(y,z)),i(i(u,i(i(i(x2,i(z,u)),i(y,i(x2,u))),v)),i(x,i(u,v))))),-4).
weight(P(i(i(x,i(y,i(i(i(z,i(u,x2)),i(u,i(z,x2))),v))),i(x,i(i(w,i(u,u)),i(w,i(y,v)))))),-4).
weight(P(i(i(x,y),i(i(z,i(y,i(i(u,u),i(x2,u)))),i(z,i(x,i(i(v,x2),i(v,u))))))),-4).
weight(P(i(i(x,i(y,z)),i(i(u,i(i(x2,i(x,z)),u)),i(i(x2,y),i(u,u))))),-4).
weight(P(i(i(x,i(i(y,i(i(z,i(u,x2)),i(u,i(z,x2)))),v)),i(i(y,i(u,u)),i(x,v)))),-4).
weight(P(i(i(x,i(i(y,y),z)),i(i(u,i(x2,i(z,u))),i(x,i(u,i(i(v,x2),i(v,u))))))),-4).
weight(P(i(i(x,i(y,i(i(z,i(i(i(u,u),i(x2,u)),u)),v))),i(i(z,x2),i(x,i(i(w,y),i(w,v)))))),-4).
weight(P(i(i(i(x,x),i(y,z)),i(i(u,y),i(i(x2,u),i(x2,i(i(z,u),u)))))),-4).
weight(P(i(i(x,i(i(i(i(y,y),i(z,u)),u),x2)),i(i(u,x),i(u,i(i(i(z,x2),v),v))))),-4).
weight(P(i(i(x,i(y,i(z,u))),i(x,i(i(i(z,i(y,u)),x2),x2)))),-4).
weight(P(i(i(x,i(y,z)),i(i(i(u,i(i(u,y),i(x,z))),x2),x2))),-4).
weight(P(i(i(x,i(y,z)),i(i(u,i(i(x2,x2),y)),i(u,i(x,z))))),-4).
weight(P(i(i(i(i(i(x,x),i(i(i(y,z),i(y,i(u,x2))),u)),u),v),i(i(u,i(z,x2)),v))),-4).
weight(P(i(i(x,i(y,i(z,u))),i(i(i(i(i(z,i(y,u)),x2),i(x,x2)),u),u))),-4).
weight(P(i(i(i(i(i(x,i(i(x,y),i(z,u))),x2),i(i(z,i(y,u)),x2)),u),u)),-4).
weight(P(i(i(x,i(y,z)),i(i(u,i(i(y,i(x,z)),x2)),i(u,x2)))),-4).
weight(P(i(i(x,i(i(i(y,z),i(i(u,i(z,x2)),i(y,i(u,x2)))),u)),i(x,u))),-4).
weight(P(i(i(x,i(y,z)),i(i(u,x),i(u,i(y,z))))),-4).
weight(P(i(i(x,i(y,z)),i(i(i(u,i(i(u,x),i(y,z))),x2),x2))),-4).
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weight(P(i(i(x,i(y,z)),i(i(u,i(i(x2,x2),x)),i(u,i(y,z))))),-4).
weight(P(i(i(i(x,i(i(y,z),z)),i(u,x2)),i(i(x,y),i(u,x2)))),-4).
weight(P(i(i(x,i(y,z)),i(i(i(u,u),i(x2,y)),i(x2,i(x,z))))),-4).
weight(P(i(i(x,i(i(y,i(z,u)),x2)),i(i(z,i(y,u)),i(x,x2)))),-4).
weight(P(i(i(i(x,x),i(y,z)),i(i(u,i(z,x2)),i(y,i(u,x2))))),-4).
weight(P(i(i(x,i(i(i(i(y,i(z,u)),x2),x2),u)),i(i(z,i(y,u)),i(x,u)))),-4).
weight(P(i(i(x,i(i(y,y),z)),i(i(u,x),i(u,z)))),-4).
weight(P(i(i(x,y),i(i(z,x),i(z,y)))),-4).  % B
weight(P(i(i(x,i(y,z)),i(x,i(y,z)))),-4).
weight(P(i(i(x,i(i(y,y),i(z,u))),i(x,i(z,u)))),-4).
weight(P(i(i(x,i(y,z)),i(y,i(x,z)))),-4).  % C
weight(P(i(i(x,y),i(x,y))),-4).
weight(P(i(i(i(x,x),i(y,z)),i(y,z))),-4).
weight(P(i(x,x)),-4).
%  Following 23/16 prove join of BCI from the 29th formula, temp.bci.halleck.out2e.
weight(P(i(i(i(i(x,y),z),u),i(x,i(i(y,z),u)))),-4).
weight(P(i(i(i(i(x,y),z),u),i(i(y,z),i(x,u)))),-4).
weight(P(i(i(x,y),i(i(z,u),i(z,i(i(u,x),y))))),-4).
weight(P(i(i(i(i(x,y),y),z),i(u,i(i(u,x),z)))),-4).
weight(P(i(x,i(i(x,i(y,z)),i(u,i(i(u,y),z))))),-4).
weight(P(i(x,i(i(x,i(y,z)),i(y,i(i(z,u),u))))),-4).
weight(P(i(i(i(i(x,y),y),z),i(u,i(i(i(v,v),i(u,x)),z)))),-4).
weight(P(i(i(i(i(x,y),y),z),i(i(i(u,u),i(v,x)),i(v,z)))),-4).
weight(P(i(i(i(x,x),i(y,i(z,u))),i(y,i(i(u,v),i(z,v))))),-4).
weight(P(i(i(i(x,x),i(y,z)),i(i(i(i(z,u),u),v),i(y,v)))),-4).
weight(P(i(i(i(x,x),i(y,z)),i(i(i(i(u,y),z),v),i(u,v)))),-4).
weight(P(i(i(x,y),i(i(i(i(y,z),z),u),i(x,u)))),-4).
weight(P(i(i(i(i(x,y),y),z),i(i(z,u),i(x,u)))),-4).
weight(P(i(i(i(i(x,i(i(i(y,z),z),u)),i(y,u)),v),i(x,v))),-4).
weight(P(i(i(i(i(i(x,x),i(y,z)),i(y,u)),v),i(i(z,u),v))),-4).
weight(P(i(i(x,i(i(i(i(i(y,z),z),u),u),v)),i(x,i(y,v)))),-4).
weight(P(i(i(x,y),i(i(z,x),i(z,y)))),-4).
weight(P(i(i(x,i(y,z)),i(y,i(x,z)))),-4).
weight(P(i(i(i(x,x),i(y,i(z,u))),i(z,i(y,u)))),-4).
weight(P(i(x,i(i(i(y,y),i(z,u)),i(i(x,z),u)))),-4).
weight(P(i(i(i(x,x),y),i(i(i(z,z),i(y,u)),u))),-4).
weight(P(i(i(i(x,x),i(i(y,i(i(y,z),z)),u)),u)),-4).
weight(P(i(x,x)),-4).
weight(P(i(i(i(u,v),w),i(i(x,i(w,y)),i(v,i(x,y))))),2).  %  Meredith's single axiom for BCK
weight(P(i(i(u,i(i(i(v,i(w,x)),i(y,i(v,x))),z)),i(w,i(u,z)))),2).
weight(P(i(u,i(i(i(v,w),x),i(w,i(i(y,i(u,z)),i(y,z)))))),2).
weight(P(i(u,i(v,i(i(u,w),i(i(x,i(v,y)),i(x,y)))))),2).
weight(P(i(u,i(i(i(i(i(i(v,w),x),i(i(y,i(x,z)),i(w,i(y,z))))),v6),i(i(v7,i(u,v8)),i(v7,v8))))),2).
weight(P(i(i(u,i(v,w)),i(x,i(i(y,i(x,z)),i(y,z))))),2).
weight(P(i(u,i(i(v,i(u,w)),i(v,w))))),2).
weight(P(i(i(u,i(i(v,i(i(w,i(v,x)),i(w,x))),y)),i(u,y))),2).
weight(P(i(i(u,i(v,w)),i(v,i(u,w))))),2).  %  C
weight(P(i(i(i(u,i(i(v,i(u,w)),i(v,w))),i(i(x,i(i(y,i(x,z)),i(y,z))),v6)),v6)),2).
weight(P(i(i(u,i(v,w)),i(i(i(x,i(i(y,i(x,z)),i(y,z))),v),i(u,w))))),2).
weight(P(i(i(u,i(i(i(i(v,i(i(w,i(v,x)),i(w,x))),y),i(z,v6)),v7)),i(i(y,v6),i(u,v7))))),2).
weight(P(i(i(u,v),i(w,i(i(i(x,i(i(y,i(x,z)),i(y,z))),u),v))))),2).
weight(P(i(i(u,v),i(i(i(w,i(i(x,i(w,y)),i(x,y))),u),v)))),2).
weight(P(i(i(i(u,i(i(v,i(u,w)),i(v,w))),i(x,y)),i(i(i(z,i(i(v6,i(z,v7)),i(v6,v7))),x),y)))),2).
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weight(P(i(i(u,i(i(i(i(v,i(i(w,i(v,x)),i(w,x))),y),z),v6)),i(i(y,z),i(u,v6)))),2).
weight(P(i(i(u,i(i(v,i(w,x)),y)),i(i(v,x),i(u,y)))),2).
weight(P(i(i(u,v),i(w,i(u,v)))),2).
weight(P(i(i(u,i(i(v,i(w,x)),y)),i(x,i(u,y)))),2).
weight(P(i(u,i(v,i(w,u)))),2).  %  K
weight(P(i(u,i(v,u))),2). % K
weight(P(i(u,i(i(i(v,w),x),i(i(y,i(x,z)),i(w,i(y,z)))))),2).
weight(P(i(i(u,v),i(w,i(i(x,i(v,y)),i(u,i(x,y)))))),2).
weight(P(i(i(u,v),i(i(w,i(v,x)),i(u,i(w,x))))),2).
weight(P(i(i(u,i(v,w)),i(i(x,v),i(x,i(u,w))))),2).
weight(P(i(i(u,v),i(u,v))),2).
weight(P(i(i(u,v),i(u,i(i(v,w),w)))),2).
weight(P(i(i(u,v),i(i(i(u,i(i(v,w),w)),x),x))),2).
weight(P(i(i(i(u,i(i(v,w),w)),x),i(i(u,v),x))),2).  %  B'
weight(P(i(i(u,v),i(i(v,w),i(u,w)))),2).  %  B'
weight(P(i(i(u,v),i(i(w,u),i(w,v)))),2).  %  B
end_of_list.

list(usable).
-P(i(x,y)) | -P(x) | P(y).
-P(i(i(a1,i(b,a2)),i(b,i(a1,a2)))) | -P(i(a1,a1)) | -P(i(i(a1,b),i(i(a2,a1),i(a2,b)))) | $ANS(all).  % BCI
end_of_list.

list(sos).
P(i(i(i(i(i(u,u),i(v,w)),i(i(x,v),w)),y),i(x,y))). % BCI-Candidate 46
%  P(i(i(x,y),i(i(i(i(z,x),y),i(i(u,u),v)),i(z,v)))).  % BCI-candidate-22
%  P(i(i(x,y),i(i(i(i(i(z,z),i(y,u)),u),x2),i(x,x2)))).  % BCI candidate 18
%  P(i(i(i(x,x),i(y,i(z,u))),i(i(u,v),i(z,i(y,v))))).  %  a 29th not among the 28 known singles for BCI.
%  P(i(i(x,i(y,z)),i(i(i(u,u),i(v,y)),i(v,i(x,z))))).  % M's BCI #1
end_of_list.

list(passive).
%  Following negs of a purported 42-step proof showing candidate 18 to be a single axiom.
-P(i(i(i(i(i(a,a),i(i(i(i(i(b,b),i(c,d)),d),a1),i(d,a1)),e)),e),f),i(i(d,c),f))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(i(d,b),i(d,i(a,c))))) | $ANS(TARG4).
-P(i(i(i(i(i(a,a),i(i(i(b,c),d),a2)),a2),d),i(i(i(i(i(e,e),i(i(i(i(i(f,f),i(c,a)),a),b),
  i(b,b)),c)),c),d),d))) | $ANS(TARG4).
-P(i(i(i(i(i(a,a),i(i(i(i(i(b,b),i(c,d)),d),a2),i(d,a2)),e)),e),f),i(i(a,b),i(a,i(i(i(c,c),
  i(i(i(d,c),f),i(b,a1))),a1))))) | $ANS(TARG4).
-P(i(i(a,b),i(a,i(i(i(c,c),i(i(i(d,i(a2,d)),i(i(e,a2),i(e,i(d,d)))),i(b,f))),f)))) | $ANS(TARG4).
-P(i(i(a,i(i(i(i(b,b),i(c,d)),d),a2)),i(a,i(i(d,c),i(d,a2))))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(i(d,a2),i(d,i(i(i(d,d),i(a2,b)),i(a,c)))))) | $ANS(TARG4).
-P(i(i(a,b),i(a,i(i(i(c,c),i(b,i(i(i(i(d,d),i(a2,d)),d),e))),i(i(f,a2),i(f,e)))))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(a,i(i(d,i(c,a2)),i(b,i(d,a2)))))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(i(d,i(i(i(a2,i(c,d)),i(b,i(a2,d))),e)),i(a,i(d,e))))) | $ANS(TARG4).
-P(i(i(a,i(b,i(i(i(c,i(d,a2)),i(d,i(c,a2))),e))),i(a,i(i(f,i(d,d)),i(f,i(b,e)))))) | $ANS(TARG4).
-P(i(i(a,b),i(i(c,i(b,i(i(d,d),i(a2,d))))),i(c,i(a,i(i(e,a2),i(e,d))))))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(i(d,i(i(a2,i(a,c)),d)),i(i(a2,b),i(d,d))))) | $ANS(TARG4).
-P(i(i(a,i(i(b,i(i(c,i(d,a2)),i(d,i(c,a2))))),e)),i(i(b,i(d,d)),i(a,e)))) | $ANS(TARG4).
-P(i(i(a,i(i(b,b),c)),i(i(d,i(a2,i(c,d))),i(a,i(d,i(i(e,a2),i(e,d))))))) | $ANS(TARG4).
-P(i(i(a,i(b,i(i(c,i(i(i(d,d),i(a2,d)),d)),e))),i(i(c,a2),i(a,i(i(f,b),i(f,e)))))) | $ANS(TARG4).
-P(i(i(i(a,a),i(b,c)),i(i(d,b),i(i(a2,d),i(a2,i(i(c,d),d)))))) | $ANS(TARG4).
-P(i(i(a,i(i(i(i(b,b),i(c,d)),d),a2)),i(i(d,a),i(d,i(i(i(c,a2),e),e))))) | $ANS(TARG4).
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-P(i(i(a,i(b,i(c,d))),i(a,i(i(i(c,i(b,d)),a2),a2)))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(i(i(d,i(i(d,b),i(a,c))),a2),a2))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(i(d,i(i(i(a2,a2),b)),i(d,i(a,c))))) | $ANS(TARG4).
-P(i(i(i(i(i(a,a),i(i(i(b,c),i(b,i(d,a2))),d)),d),e),i(i(d,i(c,a2)),e))) | $ANS(TARG4).
-P(i(i(a,i(b,i(c,d))),i(i(i(i(c,i(b,d)),a2),i(a,a2)),d),d))) | $ANS(TARG4).
-P(i(i(i(i(i(a,i(i(a,b),i(c,d))),a2),i(i(c,i(b,d)),a2)),d),d)) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(i(d,i(i(b,i(a,c)),a2)),i(d,a2)))) | $ANS(TARG4).
-P(i(i(a,i(i(i(b,c),i(i(d,i(c,a2)),i(b,i(d,a2)))),d)),i(a,d))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(i(d,a),i(d,i(b,c))))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(i(i(d,i(i(d,a),i(b,c))),a2),a2))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(i(d,i(i(a2,a2),a)),i(d,i(b,c))))) | $ANS(TARG4).
-P(i(i(i(a,i(i(b,c),c)),i(d,a2)),i(i(a,b),i(d,a2)))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(i(i(d,d),i(a2,b)),i(a2,i(a,c))))) | $ANS(TARG4).
-P(i(i(a,i(i(b,i(c,d)),a2)),i(i(c,i(b,d)),i(a,a2)))) | $ANS(TARG4).
-P(i(i(i(a,a),i(b,c)),i(i(d,i(c,a2)),i(b,i(d,a2))))) | $ANS(TARG4).
-P(i(i(a,i(i(i(i(b,i(c,d)),a2),a2),d)),i(i(c,i(b,d)),i(a,d)))) | $ANS(TARG4).
-P(i(i(a,i(i(b,b),c)),i(i(d,a),i(d,c)))) | $ANS(TARG4).
-P(i(i(a,b),i(i(c,a),i(c,b)))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(a,i(b,c)))) | $ANS(TARG4).
-P(i(i(a,i(i(b,b),i(c,d))),i(a,i(c,d)))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(b,i(a,c)))) | $ANS(TARG4).
-P(i(i(a,b),i(a,b))) | $ANS(TARG4).
-P(i(i(i(a,a),i(b,c)),i(b,c))) | $ANS(TARG4).
-P(i(a,a)) | $ANS(TARG4).
% Following negs of 72 singles, all of the known, for BCI.
-P(i(i(p,i(q,r)),i(i(i(s,s),i(t,q)),i(t,i(p,r))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(i(s,i(r,t)),i(q,i(s,t))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(r,r),i(i(i(q,s),s),t)),i(p,t)))) | $ANS(TARG2).
-P(i(i(p,q),i(i(q,i(i(r,r),i(s,t))),i(s,i(p,t))))) | $ANS(TARG2).
-P(i(i(i(i(i(p,p),i(q,r)),r),s),i(i(s,t),i(q,t)))) | $ANS(TARG2).
-P(i(i(i(i(p,q),q),r),i(i(i(s,s),i(r,t)),i(p,t)))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(i(i(i(i(s,s),p),r),t),i(q,t)))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(i(i(s,s),i(r,t)),i(q,i(p,t))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(i(i(q,r),r),s)),i(i(s,t),i(q,t)))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(s,i(i(r,i(s,t)),i(q,t))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(i(i(i(r,s),s),t),i(q,t)))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(i(r,i(s,t)),i(s,i(q,t))))) | $ANS(TARG2).
-P(i(i(p,i(i(q,q),i(r,s))),i(i(s,t),i(r,i(p,t))))) | $ANS(TARG2).
-P(i(i(p,i(i(q,q),r)),i(s,i(i(r,i(s,t)),i(p,t))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(r,r),i(q,i(s,t))),i(s,i(p,t))))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(q,i(i(i(s,s),i(r,t)),i(p,t))))) | $ANS(TARG2).
-P(i(p,i(i(i(q,q),i(r,s)),i(i(s,i(p,t)),i(r,t))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(p,r)),i(i(i(s,s),i(r,t)),i(q,t))))) | $ANS(TARG2).
-P(i(i(p,q),i(r,i(i(i(s,s),i(q,i(r,t))),i(p,t))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(i(i(i(s,t),t),q),i(s,r)))) | $ANS(TARG2).
-P(i(p,i(i(i(q,q),i(r,i(p,s))),i(i(t,r),i(t,s))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,i(r,s))),i(i(t,q),i(r,i(t,s))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(r,r),i(i(i(s,p),q),t)),i(s,t)))) | $ANS(TARG2).
-P(i(i(p,i(i(q,q),i(r,s))),i(i(t,p),i(r,i(t,s))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(i(i(i(s,q),r),t),i(s,t)))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(i(s,i(t,q)),i(t,i(s,r))))) | $ANS(TARG2).
-P(i(i(p,i(i(q,q),r)),i(i(r,i(s,t)),i(s,i(p,t))))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(i(i(s,s),i(t,p)),i(q,i(t,r))))) | $ANS(TARG2).

-P(i(i(i(p,p),i(i(i(q,r),i(i(s,q),r)),t)),i(s,t))) | $ANS(TARG2).
-P(i(i(i(p,p),i(i(i(q,r),r),s)),i(i(t,q),i(t,s)))) | $ANS(TARG2).
-P(i(i(i(p,p),i(i(i(q,r),s),t)),i(i(r,s),i(q,t)))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,i(r,s))),i(i(s,t),i(r,i(q,t))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,i(r,s))),i(i(t,r),i(t,i(q,s))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,i(r,s))),i(r,i(i(s,t),i(q,t))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,i(r,s))),i(r,i(i(t,q),i(t,s))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,i(r,s))),i(t,i(i(t,r),i(q,s))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(q,i(i(s,i(r,t)),i(s,t))))) | $ANS(TARG2).
-P(i(i(i(p,p),q),i(i(i(i(q,i(r,s)),s),t),i(r,t)))) | $ANS(TARG2).
-P(i(i(p,i(i(q,q),i(r,s))),i(i(t,r),i(t,i(p,s))))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(i(s,i(i(t,t),p)),i(q,i(s,r))))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(q,i(i(i(s,s),i(t,p)),i(t,r))))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(s,i(i(i(t,t),i(s,q)),i(p,r))))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(q,i(i(s,i(i(t,t),p)),i(s,r))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(i(i(r,r),i(s,t)),t),p),i(s,q)))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(i(q,r),r),i(i(s,s),t)),i(p,t)))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(i(r,p),q),i(i(s,s),t)),i(r,t)))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(r,r),i(s,i(q,t))),i(p,i(s,t))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(r,r),i(s,i(t,p))),i(t,i(s,q))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(r,i(i(s,s),i(q,t))),i(p,i(r,t))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(r,i(i(s,s),i(t,p))),i(t,i(r,q))))) | $ANS(TARG2).
-P(i(i(p,q),i(p,i(i(i(r,r),i(s,i(q,t))),i(s,t))))) | $ANS(TARG2).
-P(i(i(p,q),i(r,i(i(i(s,s),i(t,i(r,p))),i(t,q))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(i(r,r),i(p,s))),i(i(s,t),i(q,t))))) | $ANS(TARG2).
-P(i(p,i(i(i(q,q),i(r,i(p,s))),i(i(s,t),i(r,t))))) | $ANS(TARG2).
-P(i(p,i(i(i(q,q),i(r,i(s,t))),i(i(p,s),i(r,t))))) | $ANS(TARG2).
-P(i(p,i(i(i(q,q),i(r,s)),i(i(t,i(p,r)),i(t,s))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(i(r,r),i(p,s))),i(i(t,q),i(t,s))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(i(r,r),i(s,t))),i(i(p,s),i(q,t))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(i(r,r),s)),i(i(s,i(p,t)),i(q,t))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(p,r)),i(i(i(s,s),i(t,q)),i(t,r))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(r,s)),i(i(i(t,t),i(p,r)),i(q,s))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(p,r)),i(i(s,i(i(t,t),q)),i(s,r))))) | $ANS(TARG2).
-P(i(p,i(i(p,q),i(i(i(r,r),i(s,i(q,t))),i(s,t))))) | $ANS(TARG2).
-P(i(p,i(i(q,r),i(i(i(s,s),i(r,i(p,t))),i(q,t))))) | $ANS(TARG2).
-P(i(p,i(i(p,q),i(i(r,i(i(s,s),i(q,t))),i(r,t))))) | $ANS(TARG2).
-P(i(p,i(i(q,r),i(i(r,i(i(s,s),i(p,t))),i(q,t))))) | $ANS(TARG2).
-P(i(p,i(i(q,r),i(i(s,i(i(t,t),i(p,q))),i(s,r))))) | $ANS(TARG2).
-P(i(i(i(i(i(p,p),i(q,r)),r),s),i(i(t,q),i(t,s)))) | $ANS(TARG2).
-P(i(i(i(i(i(p,p),q),r),s),i(i(q,i(t,r)),i(t,s)))) | $ANS(TARG2).
-P(i(i(i(i(p,i(q,r)),r),s),i(i(i(t,t),p),i(q,s)))) | $ANS(TARG2).
-P(i(i(i(i(p,q),q),r),i(i(i(s,s),i(t,p)),i(t,r)))) | $ANS(TARG2).
-P(i(i(i(i(p,q),r),s),i(i(i(t,t),i(q,r)),i(p,s)))) | $ANS(TARG2).
% Following 28 are single axioms for BCI.
-P(i(i(a1,i(a2,a3)),i(i(i(a4,a4),i(a5,a2)),i(a5,i(a1,a3))))) | $ANS(TARG).
-P(i(i(i(a1,a1),i(a2,a3)),i(i(a4,i(a3,a5)),i(a2,i(a4,a5))))) | $ANS(TARG).
-P(i(i(a1,a2),i(i(i(a3,a3),i(i(i(a2,a4),a4),a5)),i(a1,a5)))) | $ANS(TARG).
-P(i(i(a1,a2),i(i(a2,i(i(a3,a3),i(a4,a5))),i(a4,i(a1,a5))))) | $ANS(TARG).
-P(i(i(i(i(i(a1,a1),i(a2,a3)),a3),a4),i(i(a4,a5),i(a2,a5)))) | $ANS(TARG).
-P(i(i(i(i(i(a1,a2),a2),a3),i(i(i(a4,a4),i(a3,a5)),i(a1,a5)))) | $ANS(TARG).
-P(i(i(a1,i(a2,a3)),i(i(i(i(i(a4,a4),a1),a3),a5),i(a2,a5)))) | $ANS(TARG).
-P(i(i(a1,i(a2,a3)),i(i(i(a4,a4),i(a3,a5)),i(a2,i(a1,a5))))) | $ANS(TARG).

```
-P(i(i(i(a1,a1),i(i(i(a2,a3),a3),a4)),i(i(a4,a5),i(a2,a5)))) | $ANS(TARG).
-P(i(i(i(a1,a1),i(a2,a3)),i(a4,i(i(a3,i(a4,a5)),i(a2,a5))))) | $ANS(TARG).
-P(i(i(i(a1,a1),i(a2,a3)),i(i(i(i(a3,a4),a4),a5),i(a2,a5)))) | $ANS(TARG).
-P(i(i(i(a1,a1),i(a2,a3)),i(i(a3,i(a4,a5)),i(a4,i(a2,a5))))) | $ANS(TARG).
-P(i(i(a1,i(i(a2,a2),i(a3,a4))),i(i(a4,a5),i(a3,i(a1,a5))))) | $ANS(TARG).
-P(i(i(a1,i(i(a2,a2),a3)),i(a4,i(i(a3,i(a4,a5)),i(a1,a5))))) | $ANS(TARG).
-P(i(i(a1,a2),i(i(i(a3,a3),i(a2,i(a4,a5))),i(a4,i(a1,a5))))) | $ANS(TARG).
-P(i(i(a1,i(a2,a3)),i(a2,i(i(i(a4,a4),i(a3,a5)),i(a1,a5))))) | $ANS(TARG).
-P(i(a1,i(i(i(a2,a2),i(a3,a4)),i(i(a4,i(a1,a5)),i(a3,a5))))) | $ANS(TARG).
-P(i(a1,i(i(a2,i(a1,a3)),i(i(i(a4,a4),i(a3,a5)),i(a2,a5))))) | $ANS(TARG).
-P(i(i(a1,a2),i(a3,i(i(i(a4,a4),i(a2,i(a3,a5))),i(a1,a5))))) | $ANS(TARG).
-P(i(i(i(a1,a1),i(a2,a3)),i(i(i(i(a4,a5),a5),a2),i(a4,a3)))) | $ANS(TARG).
-P(i(a1,i(i(i(a2,a2),i(a3,i(a1,a4))),i(i(a5,a3),i(a5,a4))))) | $ANS(TARG).
-P(i(i(i(a1,a1),i(a2,i(a3,a4))),i(i(a5,a2),i(a3,i(a5,a4))))) | $ANS(TARG).
-P(i(i(a1,a2),i(i(i(a3,a3),i(i(i(a4,a1),a2),a5)),i(a4,a5)))) | $ANS(TARG).
-P(i(i(a1,i(i(a2,a2),i(a3,a4))),i(i(a5,a1),i(a3,i(a5,a4))))) | $ANS(TARG).
-P(i(i(i(a1,a1),i(a2,a3)),i(i(i(i(a4,a2),a3),a5),i(a4,a5)))) | $ANS(TARG).
-P(i(i(i(a1,a1),i(a2,a3)),i(i(a4,i(a5,a2)),i(a5,i(a4,a3))))) | $ANS(TARG).
-P(i(i(a1,i(i(a2,a2),a3)),i(i(a3,i(a4,a5)),i(a4,i(a1,a5))))) | $ANS(TARG).
-P(i(i(a1,i(a2,a3)),i(i(i(a4,a4),i(a5,a1)),i(a2,i(a5,a3))))) | $ANS(TARG).
-P(i(a,a)) | $ANS(I).
-P(i(i(a1,i(b,a2)),i(b,i(a1,a2)))) | $ANS(C).  %  C
-P(i(i(a1,b),i(i(a2,a1),i(a2,b)))) | $ANS(B).  % B
end_of_list.

list(demodulators).
%(P(i(i(i(i(x,x),i(y,z)),i(y,u)),v),i(i(z,u),v))) = junk).
%  %  (i(i(x,x),y) = junk).
%  %  (i(y,i(x,x)) = junk).
(i(x,junk) = junk).
(i(junk,x) = junk).
(P(junk) = $T).
end_of_list.
```

A bit of commentary on this input file might be helpful.

The following three commands, when used, often enable a program to delve much deeper into the search space.

```
assign(max_weight,48).
assign(change_limit_after,800).
assign(new_max_weight,33).
```

When studying equivalential, McCune recognized that, sometimes, near the beginning of a proof, a very complex formula might be needed. At the same time, he noted that after such a formula is deduced and retained, more than occasionally, the retention of new information should be restricted further by reducing the value assigned to the max_weight. Therefore, you see what the given three items do. Next, to enable OTTER to explore deeper into the search space in the presence of a breadth-first search, I assigned the value 8 to max_distinct_vars, limiting the retention of new information by discarding any new item that required more than eight distinct variables in its representation. Use of the given input file, as it turned out, played a role in what occurred next. Indeed, I offer yet another input file.

An Input File That Led to a Contribution to Logic

```
set(hyper_res).
```

```
assign(max_weight,48).
assign(change_limit_after,800).
assign(new_max_weight,33).
assign(max_proofs,-1).
clear(print_kept).
%  set(ancestor_subsume).
set(back_sub).
% clear(for_sub).
clear(print_back_sub).
clear(print_kept).
clear(print_new_demod).
clear(print_back_demod).
clear(print_back_sub).
assign(max_distinct_vars,10).
%  assign(pick_given_ratio,2).
assign(max_mem,750000).
%  assign(max_seconds,2).
assign(report,5400).
set(order_history).
set(input_sos_first).
set(sos_queue).
assign(bsub_hint_wt,1).
set(keep_hint_subsumers).
set(process_input).

weight_list(pick_and_purge).
%  Following 32 of 33 prove cand22 a single, deriving the join of BCI, from Halleck 04-13-09.
weight(P(i(i(x,y),i(i(i(i(z,x),y),i(i(u,u),v)),i(z,v)))),-6).  % BCI-candidate-22
weight(P(i(i(i(i(v41,i(v42,v43)),i(i(i(i(v44,v42),v43),i(i(v45,v45),v46)),i(v44,v46))),
  i(i(v47,v47),v4748)),i(v41,v4748))),-6).
weight(P(i(i(i(i(v41,i(i(i(v42,i(v43,v44)),i(i(i(i(v45,v43),v44),i(i(v46,v46),v47)),i(v45,v47))),
  i(i(v4748,v4748),x))),i(v42,x)),i(i(y,y),z)),i(v41,z))),-6).
weight(P(i(i(i(v41,v41),i(i(v42,v42),i(v43,v44))),i(v43,v44))),-6).
weight(P(i(v41,v41)),-6).
weight(P(i(v41,i(i(v41,i(v42,v43)),i(i(i(i(v44,v42),v43),i(i(v45,v45),v46)),i(v44,v46))))),-6).
weight(P(i(v41,i(i(i(i(v42,i(v41,i(v43,v44))),i(i(i(i(v45,v43),v44),i(i(v46,v46),v47)),i(v45,v47))),
  i(i(v4748,v4748),x)),i(v42,x)))),-6).
weight(P(i(i(i(i(v41,i(i(i(v42,v43),i(i(i(i(v44,v42),v43),i(i(v45,v45),v46)),i(v44,v46))),
  i(v47,v4748))),i(i(i(i(x,v47),v4748),i(i(y,y),z)),i(x,z))),i(i(v4749,v4749),v4750)),i(v41,v4750))),-6).
weight(P(i(i(i(i(i(i(v41,i(v42,v43)),i(i(i(i(v44,v42),v43),i(i(v45,v45),v46)),i(v44,v46))),
  i(i(v47,v47),v4748)),i(v41,v4748)),i(x,y)),i(i(i(i(z,x),y),i(i(v4749,v4749),v4750)),i(z,v4750)))),-6).
weight(P(i(i(i(i(i(i(v41,i(v42,v43)),i(i(i(i(v44,v42),v43),i(i(v45,v45),v46)),i(v44,v46))),
  i(v47,v4748)),i(i(i(i(x,v47),v4748),i(i(y,y),z)),i(x,z))),i(i(v4749,v4749),i(i(v4750,v4750),v4751))),
  i(v41,v4751))),-6).
weight(P(i(i(v41,v42),i(v43,i(i(v43,v41),v42)))),-6).
weight(P(i(i(i(v41,v41),i(v42,v43)),i(i(i(i(v44,v42),v43),i(i(v45,v45),v46)),i(v44,v46)))),-6).
weight(P(i(i(i(i(v41,i(i(i(v42,v43),v43),i(i(v44,v44),v45))),i(v42,v45)),i(i(v46,v46),v47)),i(v41,v47))),-6).
weight(P(i(v41,i(i(v41,i(i(i(v42,v43),v43),i(i(v44,v44),v45))),i(v42,v45)))),-6).
weight(P(i(i(i(i(v41,v42),i(i(v42,i(i(i(v43,v44),v44),i(i(v45,v45),v46))),i(v43,v46))),
  i(i(v47,v47),v4748)),i(v41,v4748))),-6).
weight(P(i(i(v41,i(i(i(v42,v43),v43),i(i(v44,v44),v45))),i(v41,i(v42,v45)))),-6).
weight(P(i(i(i(i(v41,v42),v42),v43),i(v41,v43))),-6).
weight(P(i(v41,i(i(v41,i(i(i(v42,v43),v43),v44)),i(v42,v44)))),-6).
```

```
weight(P(i(v41,i(i(v41,i(i(i(v42,i(v43,v44)),i(i(i(i(v45,v43),v44),i(i(v46,v46),v47)),
  i(v45,v47))),i(i(v4748,v4748),x))),i(v42,x)))),-6).
weight(P(i(i(i(v41,i(i(i(i(v42,i(v41,i(v43,v44))),i(i(i(i(v45,v43),v44),i(i(v46,v46),v47)),
  i(v45,v47))),i(i(v4748,v4748),x)),i(v42,x))),i(i(i(y,i(z,v4749)),i(i(i(i(v4750,z),v4749),
  i(i(v4751,v4751),v4752)),i(v4750,v4752))),i(i(v4753,v4753),v4754))),i(y,v4754))),-6).
weight(P(i(i(i(i(v41,i(i(v42,i(i(v42,i(i(i(v43,i(v44,v45)),i(i(i(i(v46,v44),v45),i(i(v47,v47),v4748)),
  i(v46,v4748))),i(i(x,x),y))),i(v43,y))),i(z,v4749))),i(i(i(i(v4750,z),v4749),i(i(v4751,v4751),v4752)),
  i(v4750,v4752))),i(i(v4753,v4753),v4754)),i(v41,v4754))),-6).
weight(P(i(i(i(i(v41,v42),v43),i(i(v44,v44),v45)),i(i(v42,v43),i(v41,v45)))),-6).
weight(P(i(i(v41,v42),i(i(v43,v41),i(v43,v42)))),-6).
weight(P(i(i(i(v41,i(i(v41,i(v42,v43)),i(i(i(i(v44,v42),v43),i(i(v45,v45),v46)),i(v44,v46)))),
  i(v47,v4748)),i(i(i(i(x,v47),v4748),i(i(y,y),z)),i(x,z)))),-6).
weight(P(i(i(i(i(v41,i(i(i(i(v42,i(i(v42,i(v43,v44)),i(i(i(i(v45,v43),v44),i(i(v46,v46),v47)),
  i(v45,v47)))),i(v4748,x)),i(i(i(i(y,v4748),x),i(i(z,z),v4749)),i(y,v4749))),i(v4750,v4751))),
  i(i(i(i(v4752,v4750),v4751),i(i(v4753,v4753),v4754)),i(v4752,v4754))),i(i(v4755,v4755),v4756)),i(v41,v4756))),-6).
weight(P(i(i(i(v41,i(i(v41,i(i(i(v42,i(v43,v44)),i(i(i(i(v45,v43),v44),i(i(v46,v46),v47)),
  i(v45,v47))),i(i(v4748,v4748),x))),i(v42,x))),i(i(i(y,i(z,v4749)),i(i(i(i(v4750,z),v4749),
  i(i(v4751,v4751),v4752)),i(v4750,v4752))),i(i(v4753,v4753),v4754))),i(y,v4754))),-6).
weight(P(i(i(v41,i(v42,v43)),i(i(v44,v41),i(v44,i(v42,v43))))),-6).
weight(P(i(i(i(i(i(i(i(v41,i(v42,v43)),i(i(i(i(v44,v42),v43),i(i(v45,v45),v46)),i(v44,v46))),
  i(i(v47,v47),v4748)),i(v41,v4748)),i(x,y)),i(i(i(i(z,x),y),i(i(v4749,v4749),v4750)),
  i(z,v4750))),i(i(i(v4751,i(v4752,v4753)),i(i(i(i(v4754,v4752),v4753),i(i(v4755,v4755),v4756)),
  i(v4754,v4756))),i(i(x1,x1),y1))),i(v4751,y1))),-6).
weight(P(i(i(i(i(v41,v42),v42),i(v43,v44)),i(v41,i(v43,v44)))),-6).
weight(P(i(v41,i(i(v42,i(v41,i(v43,v44))),i(v42,i(v43,v44))))),-6).
weight(P(i(v41,i(i(v42,i(v41,v43)),i(v42,v43)))),-6).
weight(P(i(i(v41,i(i(v42,i(i(v43,i(v42,v44)),i(v43,v44))),v45)),i(v41,v45))),-6).
weight(P(i(i(v41,i(v42,v43)),i(v42,i(v41,v43)))),-6).
% Following 42 apparently prove candidate 18 a new single axiom, from Halleck.
weight(P(i(i(x,y),i(i(i(i(z,z),i(y,u)),u),x2),i(x,x2)))),-4).  % BCI candidate 18
weight(P(i(i(i(i(i(x,x),i(i(i(i(i(i(y,y),i(z,u)),u),x1),i(u,x1)),v)),v),w),i(i(u,z),w))),-4).
weight(P(i(i(x,i(y,z)),i(i(u,y),i(u,i(x,z))))),-4).
weight(P(i(i(i(i(i(x,x),i(i(i(y,z),u),x2)),x2),u),i(i(i(i(i(v,v),i(i(i(i(i(w,w),i(z,x)),x),y),
  i(y,y)),z)),z),u),u))),-4).
weight(P(i(i(i(i(i(x,x),i(i(i(i(i(i(y,y),i(z,u)),u),x2),i(u,x2)),v)),v),w),i(i(x,y),
  i(x,i(i(i(z,z),i(i(i(u,z),w),i(y,x1))),x1))))),-4).
weight(P(i(i(x,y),i(x,i(i(i(z,z),i(i(i(u,i(x2,u)),i(i(v,x2),i(v,i(u,u)))),i(y,w))),w)))),-4).
weight(P(i(i(x,i(i(i(i(y,y),i(z,u)),u),x2)),i(x,i(i(u,z),i(u,x2))))),-4).
weight(P(i(i(x,i(y,z)),i(i(u,x2),i(u,i(i(i(u,u),i(x2,y)),i(x,z)))))),-4).
weight(P(i(i(x,y),i(x,i(i(i(z,z),i(y,i(i(i(i(u,u),i(x2,u)),u),v))),i(i(w,x2),i(w,v)))))),-4).
weight(P(i(i(x,i(y,z)),i(x,i(i(u,i(z,x2)),i(y,i(u,x2)))))),-4).
weight(P(i(i(x,i(y,z)),i(i(u,i(i(i(x2,i(z,u)),i(y,i(x2,u))),v)),i(x,i(u,v))))),-4).
weight(P(i(i(x,i(y,i(i(i(z,i(u,x2)),i(u,i(z,x2))),v))),i(x,i(i(w,i(u,u)),i(w,i(y,v)))))),-4).
weight(P(i(i(x,y),i(i(z,i(y,i(i(u,u),i(x2,u)))),i(z,i(x,i(i(v,x2),i(v,u))))))),-4).
weight(P(i(i(x,i(y,z)),i(i(u,i(i(x2,i(x,z)),u)),i(i(x2,y),i(u,u))))),-4).
weight(P(i(i(x,i(i(y,i(i(z,i(u,x2)),i(u,i(z,x2)))),v)),i(i(y,i(u,u)),i(x,v)))),-4).
weight(P(i(i(x,i(i(y,y),z)),i(i(u,i(x2,i(z,u))),i(x,i(u,i(i(v,x2),i(v,u))))))),-4).
weight(P(i(i(x,i(y,i(i(z,i(i(i(u,u),i(x2,u)),u)),v))),i(i(z,x2),i(x,i(i(w,y),i(w,v)))))),-4).
weight(P(i(i(i(x,x),i(y,z)),i(i(u,y),i(i(x2,u),i(x2,i(i(z,u),u)))))),-4).
weight(P(i(i(x,i(i(i(i(y,y),i(z,u)),u),x2)),i(i(u,x),i(u,i(i(i(z,x2),v),v))))),-4).
weight(P(i(i(x,i(y,i(z,u))),i(x,i(i(i(z,i(y,u)),x2),x2)))),-4).
weight(P(i(i(x,i(y,z)),i(i(i(u,i(i(u,y),i(x,z))),x2),x2))),-4).
weight(P(i(i(x,i(y,z)),i(i(u,i(i(x2,x2),y)),i(u,i(x,z))))),-4).
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weight(P(i(i(i(i(i(x,x),i(i(i(y,z),i(y,i(u,x2))),u)),u),v),i(i(u,i(z,x2)),v))),-4).
weight(P(i(i(x,i(y,i(z,u))),i(i(i(i(i(z,i(y,u)),x2),i(x,x2)),u),u))),-4).
weight(P(i(i(i(i(i(x,i(i(x,y),i(z,u))),x2),i(i(z,i(y,u)),x2)),u),u)),-4).
weight(P(i(i(x,i(y,z)),i(i(u,i(i(y,i(x,z)),x2)),i(u,x2)))),-4).
weight(P(i(i(x,i(i(i(y,z),i(i(u,i(z,x2)),i(y,i(u,x2)))),u)),i(x,u))),-4).
weight(P(i(i(x,i(y,z)),i(i(u,x),i(u,i(y,z))))),-4).
weight(P(i(i(x,i(y,z)),i(i(i(u,i(i(u,x),i(y,z))),x2),x2))),-4).
weight(P(i(i(x,i(y,z)),i(i(u,i(i(x2,x2),x)),i(u,i(y,z))))),-4).
weight(P(i(i(i(x,i(i(y,z),z)),i(u,x2)),i(i(x,y),i(u,x2)))),-4).
weight(P(i(i(x,i(y,z)),i(i(i(u,u),i(x2,y)),i(x2,i(x,z))))),-4).
weight(P(i(i(x,i(i(y,i(z,u)),x2)),i(i(z,i(y,u)),i(x,x2)))),-4).
weight(P(i(i(i(x,x),i(y,z)),i(i(u,i(z,x2)),i(y,i(u,x2))))),-4).
weight(P(i(i(x,i(i(i(i(y,i(z,u)),x2),x2),u)),i(i(z,i(y,u)),i(x,u)))),-4).
weight(P(i(i(x,i(i(y,y),z)),i(i(u,x),i(u,z)))),-4).
weight(P(i(i(x,y),i(i(z,x),i(z,y)))),-4).  % B
weight(P(i(i(x,i(y,z)),i(x,i(y,z)))),-4).
weight(P(i(i(x,i(i(y,y),i(z,u))),i(x,i(z,u)))),-4).
weight(P(i(i(x,i(y,z)),i(y,i(x,z)))),-4).  % C
weight(P(i(i(x,y),i(x,y))),-4).
weight(P(i(i(i(x,x),i(y,z)),i(y,z))),-4).
weight(P(i(x,x)),-4).
%  Following 23/16 prove join of BCI from the 29th odd one, temp.bci.halleck.out2e.
weight(P(i(i(i(i(x,y),z),u),i(x,i(i(y,z),u)))),-4).
weight(P(i(i(i(i(x,y),z),u),i(i(y,z),i(x,u)))),-4).
weight(P(i(i(x,y),i(i(z,u),i(z,i(i(u,x),y))))),-4).
weight(P(i(i(i(i(x,y),y),z),i(u,i(i(u,x),z)))),-4).
weight(P(i(x,i(i(x,i(y,z)),i(u,i(i(u,y),z))))),-4).
weight(P(i(x,i(i(x,i(y,z)),i(y,i(i(z,u),u))))),-4).
weight(P(i(i(i(i(x,y),y),z),i(u,i(i(i(v,v),i(u,x)),z)))),-4).
weight(P(i(i(i(i(x,y),y),z),i(i(i(u,u),i(v,x)),i(v,z)))),-4).
weight(P(i(i(i(x,x),i(y,i(z,u))),i(y,i(i(u,v),i(z,v))))),-4).
weight(P(i(i(i(x,x),i(y,z)),i(i(i(i(z,u),u),v),i(y,v)))),-4).
weight(P(i(i(i(x,x),i(y,z)),i(i(i(i(u,y),z),v),i(u,v)))),-4).
weight(P(i(i(x,y),i(i(i(i(y,z),z),u),i(x,u)))),-4).
weight(P(i(i(i(i(x,y),y),z),i(i(z,u),i(x,u)))),-4).
weight(P(i(i(i(i(x,i(i(i(y,z),z),u)),i(y,u)),v),i(x,v))),-4).
weight(P(i(i(i(i(i(x,x),i(y,z)),i(y,u)),v),i(i(z,u),v))),-4).
weight(P(i(i(x,i(i(i(i(i(y,z),z),u),u),v)),i(x,i(y,v)))),-4).
weight(P(i(i(x,y),i(i(z,x),i(z,y)))),-4).
weight(P(i(i(x,i(y,z)),i(y,i(x,z)))),-4).
weight(P(i(i(i(x,x),i(y,i(z,u))),i(z,i(y,u)))),-4).
weight(P(i(x,i(i(i(y,y),i(z,u)),i(i(x,z),u)))),-4).
weight(P(i(i(i(x,x),y),i(i(i(z,z),i(y,u)),u))),-4).
weight(P(i(i(i(x,x),i(i(y,i(i(y,z),z)),u)),u)),-4).
weight(P(i(x,x)),-4).
%  Following 31 include Meredith's single for BCK, and his 30-step proof.
weight(P(i(i(i(u,v),w),i(i(x,i(w,y)),i(v,i(x,y))))),2).  %  Meredith's single axiom for BCK
weight(P(i(i(u,i(i(i(v,i(w,x)),i(y,i(v,x))),z)),i(w,i(u,z)))),2).
weight(P(i(u,i(i(i(v,w),x),i(w,i(i(y,i(u,z)),i(y,z)))))),2).
weight(P(i(u,i(v,i(i(u,w),i(i(x,i(v,y)),i(x,y)))))),2).
weight(P(i(u,i(i(i(i(i(i(v,w),x),i(i(y,i(x,z)),i(w,i(y,z))))),v6),i(i(v7,i(u,v8)),i(v7,v8))))),2).
weight(P(i(i(u,i(v,w)),i(x,i(i(y,i(x,z)),i(y,z))))),2).
weight(P(i(u,i(i(v,i(u,w)),i(v,w))))),2).
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weight(P(i(i(u,i(i(v,i(i(w,i(v,x)),i(w,x))),y)),i(u,y))),2).
weight(P(i(i(u,i(v,w)),i(v,i(u,w)))),2).  %  C
weight(P(i(i(i(u,i(i(v,i(u,w)),i(v,w))),i(i(x,i(i(y,i(x,z)),i(y,z))),v6)),v6)),2).
weight(P(i(i(u,i(v,w)),i(i(i(x,i(i(y,i(x,z)),i(y,z))),v),i(u,w)))),2).
weight(P(i(i(u,i(i(i(i(v,i(i(w,i(v,x)),i(w,x))),y),i(z,v6)),v7)),i(i(y,v6),i(u,v7)))),2).
weight(P(i(i(u,v),i(w,i(i(i(x,i(i(y,i(x,z)),i(y,z))),u),v)))),2).
weight(P(i(i(u,v),i(i(i(w,i(i(x,i(w,y)),i(x,y))),u),v))),2).
weight(P(i(i(i(u,i(i(v,i(u,w)),i(v,w))),i(x,y)),i(i(i(z,i(i(v6,i(z,v7)),i(v6,v7))),x),y))),2).
weight(P(i(i(u,i(i(i(i(v,i(i(w,i(v,x)),i(w,x))),y),z),v6)),i(i(y,z),i(u,v6)))),2).
weight(P(i(i(u,i(i(v,i(w,x)),y)),i(i(v,x),i(u,y)))),2).
weight(P(i(i(u,v),i(w,i(u,v)))),2).
weight(P(i(i(u,i(i(v,i(w,x)),y)),i(x,i(u,y)))),2).
weight(P(i(u,i(v,i(w,u)))),2).  %  K
weight(P(i(u,i(v,u))),2). % K
weight(P(i(u,i(i(i(v,w),x),i(i(y,i(x,z)),i(w,i(y,z))))),2).
weight(P(i(i(u,v),i(w,i(i(x,i(v,y)),i(u,i(x,y))))),2).
weight(P(i(i(u,v),i(i(w,i(v,x)),i(u,i(w,x))))),2).
weight(P(i(i(u,i(v,w)),i(i(x,v),i(x,i(u,w))))),2).
weight(P(i(i(u,v),i(u,v))),2).
weight(P(i(i(u,v),i(u,i(i(v,w),w)))),2).
weight(P(i(i(u,v),i(i(i(u,i(i(v,w),w)),x),x))),2).
weight(P(i(i(i(u,i(i(v,w),w)),x),i(i(u,v),x))),2).  %  B'
weight(P(i(i(u,v),i(i(v,w),i(u,w)))),2).  %  B'
weight(P(i(i(u,v),i(i(w,u),i(w,v)))),2).  %  B
end_of_list.

list(usable).
-P(i(x,y)) | -P(x) | P(y).
-P(i(i(a1,i(b,a2)),i(b,i(a1,a2)))) | -P(i(a1,a1)) | -P(i(i(a1,b),i(i(a2,a1),i(a2,b)))) | $ANS(all).  % BCI
end_of_list.

list(sos).
P(i(i(i(i(i(u,u),i(v,w)),i(i(x,v),w)),y),i(x,y))). % BCI-Candidate 46
%  P(i(i(x,y),i(i(i(i(z,x),y),i(i(u,u),v)),i(z,v)))).  % BCI-candidate-22
%  P(i(i(x,y),i(i(i(i(i(z,z),i(y,u)),u),x2),i(x,x2)))).  % BCI candidate 18
%  P(i(i(i(x,x),i(y,i(z,u))),i(i(u,v),i(z,i(y,v))))).  %  a 29th not among the 28 known singles for BCI.
%  P(i(i(x,i(y,z)),i(i(i(u,u),i(v,y)),i(v,i(x,z))))).  % M's BCI #1
%  Following 27 sorted to remove duplicates, temp.halleck.bci.cand46.out1k.
P(i(i(i(x,i(i(i(y,y),i(x,z)),z)),i(i(i(i(u,u),i(v,w)),i(i(v6,v),w)),v7)),i(v6,v7))).
P(i(i(i(x,i(i(x,y),y)),i(i(i(i(z,z),i(u,v)),i(i(w,u),v)),v6)),i(w,v6))).
P(i(i(i(x,i(y,i(i(x,i(y,z)),z))),i(i(u,i(v,i(i(u,i(v,w)),w))),v6)),v6)).
P(i(i(i(x,x),i(i(i(i(i(i(i(y,y),i(z,u)),i(i(v,z),u)),w),i(v,w)),v6),v6),v7)),v7)).
P(i(i(i(x,x),i(i(i(i(i(i(y,y),i(z,u)),i(i(v,z),u)),w),i(v,w)),v6)),v6)).
P(i(i(i(x,x),i(i(i(i(y,y),i(z,u)),i(i(i(v,i(i(v,w),w)),z),u)),v6)),v6)).
P(i(i(i(x,x),i(i(y,i(i(i(z,i(u,i(i(z,i(u,v)),v))),i(y,w)),w)),v6)),v6)).
P(i(i(i(x,x),i(i(y,i(i(i(z,z),i(y,u)),u)),v)),v)).
P(i(i(i(x,x),i(i(y,i(z,i(i(y,i(z,u)),u))),v)),v)).
P(i(i(i(x,x),i(y,i(i(z,z),u))),i(y,u))).
P(i(i(i(x,x),i(y,z)),i(i(i(u,i(i(i(v,v),i(u,w)),w)),y),z))).
P(i(i(i(x,x),y),i(i(i(z,z),i(y,u)),u))).
P(i(i(i(x,x),y),y)).
P(i(i(x,i(i(y,y),z)),i(x,z))).
P(i(x,i(i(i(i(i(y,y),i(i(i(i(z,z),i(u,i(i(v,v),w))),i(u,w)),v6)),v6),i(x,v7)),v7))).
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P(i(x,i(i(i(i(i(y,y),i(i(z,i(u,i(i(z,i(u,v)),v))),w)),w),i(x,v6)),v6))).
P(i(x,i(i(i(y,i(z,i(i(y,i(z,u)),u))),i(i(i(i(v,v),i(w,v6)),i(i(x,w),v6)),v7)),v7))).
P(i(x,i(i(i(y,i(z,i(i(y,i(z,u)),u))),i(x,v)),v))).
P(i(x,i(i(i(y,y),i(i(i(i(z,z),i(u,v)),i(i(x,u),v)),w)),w))).
P(i(x,i(i(i(y,y),i(i(i(x,z),z),u)),u))).
P(i(x,i(i(i(y,y),i(i(z,i(i(i(i(u,u),i(v,w)),i(i(x,v),w)),i(z,v6)),v6)),v7)),v7))).
P(i(x,i(i(i(y,y),i(i(z,i(i(x,i(z,u)),u)),v)),v))).
P(i(x,i(i(i(y,y),i(x,z)),z))).
P(i(x,i(i(x,y),y))).
P(i(x,i(y,i(i(i(i(i(z,z),i(x,u)),u),i(y,v)),v)))).
P(i(x,i(y,i(i(x,i(y,z)),z)))).
P(i(x,x)).
% Following 28 sorted proof steps from temp.halleck.bci.cand46.out1b1.
P(i(i(i(i(i(x,i(i(x,y),y)),z),z),u),u)).
P(i(i(i(i(x,y),i(z,i(i(z,x),y))),u),i(i(u,v),v))).
P(i(i(i(i(x,i(i(x,y),y)),z),z)).
P(i(i(i(i(x,i(y,i(i(x,i(y,z)),z))),i(i(u,i(i(i(v,i(w,i(i(v,i(w,v6)),v6))),i(u,v7)),v7)),v8)),v8)).
P(i(i(i(i(x,i(y,i(i(x,i(y,z)),z))),i(i(u,i(v,i(i(u,i(v,w)),w))),v6)),v6)).
P(i(i(i(i(x,x),i(i(i(i(y,y),i(z,u)),i(i(v,z),u)),w)),i(v,w))).
P(i(i(i(i(x,x),i(i(y,i(i(i(z,i(u,i(i(z,i(u,v)),v))),v))),i(y,w)),w)),v6)),v6)).
P(i(i(i(i(x,x),i(i(y,i(i(i(z,z),i(y,u)),u)),v)),v)).
P(i(i(i(i(x,x),i(i(y,y),i(z,u))),i(i(i(v,v),z),u))).
P(i(i(i(i(x,x),i(i(y,y),i(z,u))),i(v,i(i(v,z),u)))).
P(i(i(i(i(x,x),i(i(y,y),z)),z)).
P(i(i(i(i(x,x),i(y,z)),i(i(i(u,u),y),z))).
P(i(i(i(i(x,x),i(y,z)),i(u,i(i(u,y),z)))).
P(i(i(i(i(x,x),y),y)).
P(i(i(i(x,y),i(i(i(z,i(i(z,x),y)),u),u))).
P(i(i(i(x,y),i(i(i(z,z),x),y))).
P(i(i(i(x,y),i(z,i(i(z,x),y)))).
P(i(x,i(i(i(y,i(z,i(i(y,i(z,u)),u))),i(i(i(i(v,v),i(w,v6)),i(i(x,w),v6)),v7)),v7))).
P(i(x,i(i(i(y,i(z,i(i(y,i(z,u)),u))),i(x,v)),v))).
P(i(x,i(i(i(y,y),i(i(i(i(z,i(u,i(i(z,i(u,v)),v))),i(x,w)),w),v6)),v6))).
P(i(x,i(i(i(y,y),i(i(z,i(i(x,i(z,u)),u)),v)),v))).
P(i(x,i(i(i(y,y),i(x,z)),z))).
P(i(x,i(i(i(y,y),i(z,u)),i(i(x,z),u)))).
P(i(x,i(i(x,i(i(i(i(y,z),i(u,i(i(u,y),z))),v)),i(i(v,w),w)))).
P(i(x,i(i(x,y),i(i(y,z),z)))).
P(i(x,i(i(x,y),y))).
P(i(x,i(y,i(i(x,i(y,z)),z)))).
P(i(x,x)).
% following another, from temp.halleck.bci.cand46.out1b2.
P(i(i(i(x,i(y,i(i(x,i(y,z)),z))),i(i(i(i(i(u,u),i(v,w)),i(i(v6,v),w)),v7),i(v6,v7)),v8)),v8)).
% Following 6 more, from temp.halleck.bci.cand46.out1b3.
P(i(x,i(i(i(i(i(y,y),i(z,u)),i(i(x,z),u)),v),v))).
P(i(i(i(x,i(y,i(i(x,i(y,z)),z))),i(i(u,i(i(u,v),v)),w)),w)).
P(i(i(i(x,i(i(x,i(i(i(i(y,y),i(z,u)),i(i(v,z),u)),w)),i(v,w))),v6),v6)).
P(i(i(i(i(i(x,x),i(y,z)),i(i(i(i(u,i(v,i(i(u,i(v,w)),w))),i(i(v6,i(i(v6,v7),v7)),v8)),v8),y),z)),v9),v9)).
P(i(i(i(x,i(i(x,i(i(i(i(y,y),i(z,u)),i(i(v,z),u)),w)),i(v,w))),i(i(i(i(v6,v6),i(v7,v8)),
  i(i(v9,v7),v8)),v10)),i(v9,v10))).
P(i(i(x,i(i(y,y),z)),i(x,z))).
end_of_list.

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list(passive).
%  Following negs of a purported 42-step proof showing candidate 18 to be a single axiom.
-P(i(i(i(i(i(a,a),i(i(i(i(i(b,b),i(c,d)),d),a1),i(d,a1)),e)),e),f),i(i(d,c),f))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(i(d,b),i(d,i(a,c))))) | $ANS(TARG4).
-P(i(i(i(i(i(a,a),i(i(i(b,c),d),a2)),a2),d),i(i(i(i(i(e,e),i(i(i(i(i(f,f),i(c,a)),a),b),
  i(b,b)),c)),c),d),d))) | $ANS(TARG4).
-P(i(i(i(i(i(a,a),i(i(i(i(i(b,b),i(c,d)),d),a2),i(d,a2)),e)),e),f),i(i(a,b),i(a,i(i(i(c,c),
  i(i(i(d,c),f),i(b,a1))),a1))))) | $ANS(TARG4).
-P(i(i(a,b),i(a,i(i(i(c,c),i(i(i(d,i(a2,d)),i(i(e,a2),i(e,i(d,d)))),i(b,f))),f)))) | $ANS(TARG4).
-P(i(i(a,i(i(i(i(b,b),i(c,d)),d),a2)),i(a,i(i(d,c),i(d,a2))))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(i(d,a2),i(d,i(i(i(d,d),i(a2,b)),i(a,c)))))) | $ANS(TARG4).
-P(i(i(a,b),i(a,i(i(i(c,c),i(b,i(i(i(i(d,d),i(a2,d)),d),e))),i(i(f,a2),i(f,e)))))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(a,i(i(d,i(c,a2)),i(b,i(d,a2)))))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(i(d,i(i(i(a2,i(c,d)),i(b,i(a2,d))),e)),i(a,i(d,e))))) | $ANS(TARG4).
-P(i(i(a,i(b,i(i(i(c,i(d,a2)),i(d,i(c,a2))),e))),i(a,i(i(f,i(d,d)),i(f,i(b,e)))))) | $ANS(TARG4).
-P(i(i(a,b),i(i(c,i(b,i(i(d,d),i(a2,d)))),i(c,i(a,i(i(e,a2),i(e,d))))))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(i(d,i(i(a2,i(a,c)),d)),i(i(a2,b),i(d,d))))) | $ANS(TARG4).
-P(i(i(a,i(i(b,i(i(c,i(d,a2)),i(d,i(c,a2)))),e)),i(i(b,i(d,d)),i(a,e)))) | $ANS(TARG4).
-P(i(i(a,i(i(b,b),c)),i(i(d,i(a2,i(c,d))),i(a,i(d,i(i(e,a2),i(e,d))))))) | $ANS(TARG4).
-P(i(i(a,i(b,i(i(c,i(i(i(d,d),i(a2,d)),d)),e))),i(i(c,a2),i(a,i(i(f,b),i(f,e)))))) | $ANS(TARG4).
-P(i(i(i(a,a),i(b,c)),i(i(d,b),i(i(a2,d),i(a2,i(i(c,d),d)))))) | $ANS(TARG4).
-P(i(i(a,i(i(i(i(b,b),i(c,d)),d),a2)),i(i(d,a),i(d,i(i(i(c,a2),e),e))))) | $ANS(TARG4).
-P(i(i(a,i(b,i(c,d))),i(a,i(i(i(c,i(b,d)),a2),a2)))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(i(i(d,i(i(d,b),i(a,c))),a2),a2))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(i(d,i(i(a2,a2),b)),i(d,i(a,c))))) | $ANS(TARG4).
-P(i(i(i(i(i(a,a),i(i(i(b,c),i(b,i(d,a2))),d)),d),e),i(i(d,i(c,a2)),e))) | $ANS(TARG4).
-P(i(i(a,i(b,i(c,d))),i(i(i(i(c,i(b,d)),a2),i(a,a2)),d),d))) | $ANS(TARG4).
-P(i(i(i(i(i(a,i(i(a,b),i(c,d))),a2),i(i(c,i(b,d)),a2)),d),d)) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(i(d,i(i(b,i(a,c)),a2)),i(d,a2)))) | $ANS(TARG4).
-P(i(i(a,i(i(i(b,c),i(i(d,i(c,a2)),i(b,i(d,a2)))),d)),i(a,d))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(i(d,a),i(d,i(b,c))))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(i(i(d,i(i(d,a),i(b,c))),a2),a2))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(i(d,i(i(a2,a2),a)),i(d,i(b,c))))) | $ANS(TARG4).
-P(i(i(i(a,i(i(b,c),c)),i(d,a2)),i(i(a,b),i(d,a2)))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(i(i(d,d),i(a2,b)),i(a2,i(a,c))))) | $ANS(TARG4).
-P(i(i(a,i(i(b,i(c,d)),a2)),i(i(c,i(b,d)),i(a,a2)))) | $ANS(TARG4).
-P(i(i(i(a,a),i(b,c)),i(i(d,i(c,a2)),i(b,i(d,a2))))) | $ANS(TARG4).
-P(i(i(a,i(i(i(i(b,i(c,d)),a2),a2),d)),i(i(c,i(b,d)),i(a,d)))) | $ANS(TARG4).
-P(i(i(a,i(i(b,b),c)),i(i(d,a),i(d,c)))) | $ANS(TARG4).
-P(i(i(a,b),i(i(c,a),i(c,b)))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(a,i(b,c)))) | $ANS(TARG4).
-P(i(i(a,i(i(b,b),i(c,d))),i(a,i(c,d)))) | $ANS(TARG4).
-P(i(i(a,i(b,c)),i(b,i(a,c)))) | $ANS(TARG4).
-P(i(i(a,b),i(a,b))) | $ANS(TARG4).
-P(i(i(i(a,a),i(b,c)),i(b,c))) | $ANS(TARG4).
-P(i(a,a)) | $ANS(TARG4).
%  Following negs of 72 singles, all of the known, for BCI.
-P(i(i(p,i(q,r)),i(i(i(s,s),i(t,q)),i(t,i(p,r))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(i(s,i(r,t)),i(q,i(s,t))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(r,r),i(i(i(q,s),s),t)),i(p,t)))) | $ANS(TARG2).
-P(i(i(p,q),i(i(q,i(i(r,r),i(s,t))),i(s,i(p,t))))) | $ANS(TARG2).
-P(i(i(i(i(i(p,p),i(q,r)),r),s),i(i(s,t),i(q,t)))) | $ANS(TARG2).
-P(i(i(i(i(p,q),q),r),i(i(i(s,s),i(r,t)),i(p,t)))) | $ANS(TARG2).
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-P(i(i(p,i(q,r)),i(i(i(i(i(s,s),p),r),t),i(q,t)))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(i(i(s,s),i(r,t)),i(q,i(p,t))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(i(i(q,r),r),s)),i(i(s,t),i(q,t)))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(s,i(i(r,i(s,t)),i(q,t))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(i(i(i(r,s),s),t),i(q,t)))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(i(r,i(s,t)),i(s,i(q,t))))) | $ANS(TARG2).
-P(i(i(p,i(i(q,q),i(r,s))),i(i(s,t),i(r,i(p,t))))) | $ANS(TARG2).
-P(i(i(p,i(i(q,q),r)),i(s,i(i(r,i(s,t)),i(p,t))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(r,r),i(q,i(s,t))),i(s,i(p,t))))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(q,i(i(i(s,s),i(r,t)),i(p,t))))) | $ANS(TARG2).
-P(i(p,i(i(i(q,q),i(r,s)),i(i(s,i(p,t)),i(r,t))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(p,r)),i(i(i(s,s),i(r,t)),i(q,t))))) | $ANS(TARG2).
-P(i(i(p,q),i(r,i(i(i(s,s),i(q,i(r,t))),i(p,t))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(i(i(i(s,t),t),q),i(s,r)))) | $ANS(TARG2).
-P(i(p,i(i(i(q,q),i(r,i(p,s))),i(i(t,r),i(t,s))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,i(r,s))),i(i(t,q),i(r,i(t,s))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(r,r),i(i(i(s,p),q),t)),i(s,t)))) | $ANS(TARG2).
-P(i(i(p,i(i(q,q),i(r,s))),i(i(t,p),i(r,i(t,s))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(i(i(i(s,q),r),t),i(s,t)))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(i(s,i(t,q)),i(t,i(s,r))))) | $ANS(TARG2).
-P(i(i(p,i(i(q,q),r)),i(i(r,i(s,t)),i(s,i(p,t))))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(i(i(s,s),i(t,p)),i(q,i(t,r))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(i(i(q,r),i(i(s,q),r)),t)),i(s,t))) | $ANS(TARG2).
-P(i(i(i(p,p),i(i(i(q,r),r),s)),i(i(t,q),i(t,s)))) | $ANS(TARG2).
-P(i(i(i(p,p),i(i(i(q,r),s),t)),i(i(r,s),i(q,t)))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,i(r,s))),i(i(s,t),i(r,i(q,t))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,i(r,s))),i(i(t,r),i(t,i(q,s))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,i(r,s))),i(r,i(i(s,t),i(q,t))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,i(r,s))),i(r,i(i(t,q),i(t,s))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,i(r,s))),i(t,i(i(t,r),i(q,s))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(q,i(i(s,i(r,t)),i(s,t))))) | $ANS(TARG2).
-P(i(i(i(p,p),q),i(i(i(i(q,i(r,s)),s),t),i(r,t)))) | $ANS(TARG2).
-P(i(i(p,i(i(q,q),i(r,s))),i(i(t,r),i(t,i(p,s))))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(i(s,i(i(t,t),p)),i(q,i(s,r))))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(q,i(i(i(s,s),i(t,p)),i(t,r))))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(s,i(i(i(t,t),i(s,q)),i(p,r))))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(q,i(i(s,i(i(t,t),p)),i(s,r))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(i(i(r,r),i(s,t)),t),p),i(s,q)))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(i(q,r),r),i(i(s,s),t)),i(p,t)))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(i(r,p),q),i(i(s,s),t)),i(r,t)))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(r,r),i(s,i(q,t))),i(p,i(s,t))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(r,r),i(s,i(t,p))),i(t,i(s,q))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(r,i(i(s,s),i(q,t))),i(p,i(r,t))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(r,i(i(s,s),i(t,p))),i(t,i(r,q))))) | $ANS(TARG2).
-P(i(i(p,q),i(p,i(i(i(r,r),i(s,i(q,t))),i(s,t))))) | $ANS(TARG2).
-P(i(i(p,q),i(r,i(i(i(s,s),i(t,i(r,p))),i(t,q))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(i(r,r),i(p,s))),i(i(s,t),i(q,t))))) | $ANS(TARG2).
-P(i(p,i(i(i(q,q),i(r,i(p,s))),i(i(s,t),i(r,t))))) | $ANS(TARG2).
-P(i(p,i(i(i(q,q),i(r,i(s,t))),i(i(p,s),i(r,t))))) | $ANS(TARG2).
-P(i(p,i(i(i(q,q),i(r,s)),i(i(t,i(p,r)),i(t,s))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(i(r,r),i(p,s))),i(i(t,q),i(t,s))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(i(r,r),i(s,t))),i(i(p,s),i(q,t))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(i(r,r),s)),i(i(s,i(p,t)),i(q,t))))) | $ANS(TARG2).

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-P(i(p,i(i(q,i(p,r)),i(i(i(s,s),i(t,q)),i(t,r))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(r,s)),i(i(i(t,t),i(p,r)),i(q,s))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(p,r)),i(i(s,i(i(t,t),q)),i(s,r))))) | $ANS(TARG2).
-P(i(p,i(i(p,q),i(i(i(r,r),i(s,i(q,t))),i(s,t))))) | $ANS(TARG2).
-P(i(p,i(i(q,r),i(i(i(s,s),i(r,i(p,t))),i(q,t))))) | $ANS(TARG2).
-P(i(p,i(i(p,q),i(i(r,i(i(s,s),i(q,t))),i(r,t))))) | $ANS(TARG2).
-P(i(p,i(i(q,r),i(i(r,i(i(s,s),i(p,t))),i(q,t))))) | $ANS(TARG2).
-P(i(p,i(i(q,r),i(i(s,i(i(t,t),i(p,q))),i(s,r))))) | $ANS(TARG2).
-P(i(i(i(i(i(p,p),i(q,r)),r),s),i(i(t,q),i(t,s)))) | $ANS(TARG2).
-P(i(i(i(i(i(p,p),q),r),s),i(i(q,i(t,r)),i(t,s)))) | $ANS(TARG2).
-P(i(i(i(i(p,i(q,r)),r),s),i(i(i(t,t),p),i(q,s)))) | $ANS(TARG2).
-P(i(i(i(i(p,q),q),r),i(i(i(s,s),i(t,p)),i(t,r)))) | $ANS(TARG2).
-P(i(i(i(i(p,q),r),s),i(i(i(t,t),i(q,r)),i(p,s)))) | $ANS(TARG2).
%  Following 28 are single axioms for BCI.
-P(i(i(a1,i(a2,a3)),i(i(i(a4,a4),i(a5,a2)),i(a5,i(a1,a3))))) | $ANS(TARG).
-P(i(i(i(a1,a1),i(a2,a3)),i(i(a4,i(a3,a5)),i(a2,i(a4,a5))))) | $ANS(TARG).
-P(i(i(a1,a2),i(i(i(a3,a3),i(i(i(a2,a4),a4),a5)),i(a1,a5)))) | $ANS(TARG).
-P(i(i(a1,a2),i(i(a2,i(i(a3,a3),i(a4,a5))),i(a4,i(a1,a5))))) | $ANS(TARG).
-P(i(i(i(i(i(a1,a1),i(a2,a3)),a3),a4),i(i(a4,a5),i(a2,a5)))) | $ANS(TARG).
-P(i(i(i(i(a1,a2),a2),a3),i(i(i(a4,a4),i(a3,a5)),i(a1,a5)))) | $ANS(TARG).
-P(i(i(a1,i(a2,a3)),i(i(i(i(a4,a4),a1),a3),a5),i(a2,a5)))) | $ANS(TARG).
-P(i(i(a1,i(a2,a3)),i(i(i(a4,a4),i(a3,a5)),i(a2,i(a1,a5))))) | $ANS(TARG).
-P(i(i(i(a1,a1),i(i(i(a2,a3),a3),a4)),i(i(a4,a5),i(a2,a5)))) | $ANS(TARG).
-P(i(i(i(a1,a1),i(a2,a3)),i(a4,i(i(a3,i(a4,a5)),i(a2,a5))))) | $ANS(TARG).
-P(i(i(i(a1,a1),i(a2,a3)),i(i(i(i(a3,a4),a4),a5),i(a2,a5)))) | $ANS(TARG).
-P(i(i(i(a1,a1),i(a2,a3)),i(i(a3,i(a4,a5)),i(a4,i(a2,a5))))) | $ANS(TARG).
-P(i(i(a1,i(i(a2,a2),i(a3,a4))),i(i(a4,a5),i(a3,i(a1,a5))))) | $ANS(TARG).
-P(i(i(a1,i(i(a2,a2),i(a3,a4))),i(a4,i(i(a3,i(a4,a5)),i(a1,a5))))) | $ANS(TARG).
-P(i(i(a1,a2),i(i(i(a3,a3),i(a2,i(a4,a5))),i(a4,i(a1,a5))))) | $ANS(TARG).
-P(i(i(a1,i(a2,a3)),i(a2,i(i(i(a4,a4),i(a3,a5)),i(a1,a5))))) | $ANS(TARG).
-P(i(a1,i(i(i(a2,a2),i(a3,a4)),i(i(a4,i(a1,a5)),i(a3,a5))))) | $ANS(TARG).
-P(i(a1,i(i(a2,i(a1,a3)),i(i(i(a4,a4),i(a3,a5)),i(a2,a5))))) | $ANS(TARG).
-P(i(i(a1,a2),i(a3,i(i(i(a4,a4),i(a2,i(a3,a5))),i(a1,a5))))) | $ANS(TARG).
-P(i(i(i(a1,a1),i(a2,a3)),i(i(i(i(a4,a5),a5),a2),i(a4,a3)))) | $ANS(TARG).
-P(i(a1,i(i(i(a2,a2),i(a3,i(a1,a4))),i(i(a5,a3),i(a5,a4))))) | $ANS(TARG).
-P(i(i(i(a1,a1),i(a2,i(a3,a4))),i(i(a5,a2),i(a3,i(a5,a4))))) | $ANS(TARG).
-P(i(i(a1,a2),i(i(i(a3,a3),i(i(i(a4,a1),a2),a5)),i(a4,a5)))) | $ANS(TARG).
-P(i(i(a1,i(i(a2,a2),i(a3,a4))),i(i(a5,a1),i(a3,i(a5,a4))))) | $ANS(TARG).
-P(i(i(i(a1,a1),i(a2,a3)),i(i(i(i(a4,a2),a3),a5),i(a4,a5)))) | $ANS(TARG).
-P(i(i(i(a1,a1),i(a2,a3)),i(i(a4,i(a5,a2)),i(a5,i(a4,a3))))) | $ANS(TARG).
-P(i(i(a1,i(i(a2,a2),i(a3,a4))),i(i(a3,i(a4,a5)),i(a4,i(a1,a5))))) | $ANS(TARG).
-P(i(i(a1,i(a2,a3)),i(i(i(a4,a4),i(a5,a1)),i(a2,i(a5,a3))))) | $ANS(TARG).
-P(i(a,a)) | $ANS(I).
-P(i(i(a1,i(b,a2)),i(b,i(a1,a2)))) | $ANS(C).  %  C
-P(i(i(a1,b),i(i(a2,a1),i(a2,b)))) | $ANS(B).  % B
end_of_list.

list(demodulators).
%(P(i(i(i(i(i(x,x),i(y,z)),i(y,u)),v),i(i(z,u),v))) = junk).
%  %  (i(i(x,x),y) = junk).
%  %  (i(y,i(x,x)) = junk).
(i(x,junk) = junk).
(i(junk,x) = junk).
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(P(junk) = $T).
end_of_list.
```

This file was used in an experiment that was being conducted as the cited newsletter was made available. For any of you that might be suspicious, when you learn more about history, note that I had no idea what was about to occur. Indeed, not long after the newsletter hit the stand (so to speak), with the given input file, OTTER proved each of *I* (no surprise), *B*, and *C*. The program also returned a proof of a known single axiom. Therefore, from two different viewpoints, the question that had been open concerning the status of candidate 46 in the context of being a single axiom was answered in the affirmative. More than 86,000 CPU-seconds were required. Of course, since the list(sos) contained many formulas from earlier runs, being used in the spirit of lemmas, the desired proofs (relying solely on candidate 46) were still not in hand. (For the curious, the use of this last-given input file does not, or did not, return a proof of the join. The reason is that neither *B* nor *C* had yet been the focus of attention to drive the reasoning; such would have required even more time. More than 8.5 CPU-hours had been required, with the retention of more than 124,000 new conclusions.)

Therefore, what is needed is an input file that benefits from all of the foregoing experiments and returns at least one proof of a known single axiom and also yields a proof of the conjunction (join) of *B*, *C*, and *I*. The following input file, almost identical (except for the removal of comments) to that used by me shortly after the newsletter was published, yields the sought-after proofs, of a single axiom (for Ulrich) and of the conjunction (for Halleck).

An Input File That Answers in the Affirmative the Question Concerning Candidate 46

```
set(hyper_res).
assign(max_weight,48).
%  assign(change_limit_after,800).
%  assign(new_max_weight,33).
assign(max_proofs,16).
clear(print_kept).
set(ancestor_subsume).
set(back_sub).
% clear(for_sub).
clear(print_back_sub).
clear(print_kept).
clear(print_new_demod).
clear(print_back_demod).
clear(print_back_sub).
assign(max_distinct_vars,14).
assign(pick_given_ratio,2).
assign(max_mem,750000).
%  assign(max_seconds,2).
assign(report,5400).
set(order_history).
set(input_sos_first).
%  set(sos_queue).
assign(bsub_hint_wt,1).
set(keep_hint_subsumers).
%  set(process_input).

weight_list(pick_and_purge).
%  Following is the typical axiomatization of BCI.
weight(P(i(i(x,y),i(i(z,x),i(z,y)))),-2).  % B
weight(P(i(i(x,i(y,z)),i(y,i(x,z)))),-2).  % C
```

```
weight(P(i(x,x)),-2).  % I
%  Following 27 sorted to remove duplicates, temp.halleck.bci.cand46.out1k.
weight(P(i(i(i(x,i(i(i(y,y),i(x,z)),z)),i(i(i(i(u,u),i(v,w)),i(i(v6,v),w)),v7)),i(v6,v7))),1).
weight(P(i(i(i(x,i(i(x,y),y)),i(i(i(i(z,z),i(u,v)),i(i(w,u),v)),v6)),i(w,v6))),1).
weight(P(i(i(i(x,i(y,i(i(x,i(y,z)),z))),i(i(u,i(v,i(i(u,i(v,w)),w))),v6)),v6)),1).
weight(P(i(i(i(x,x),i(i(i(i(i(i(i(y,y),i(z,u)),i(i(v,z),u)),w),i(v,w)),v6),v6),v7)),v7)),1).
weight(P(i(i(i(x,x),i(i(i(i(i(i(y,y),i(z,u)),i(i(v,z),u)),w),i(v,w)),v6)),v6)),1).
weight(P(i(i(i(x,x),i(i(i(i(y,y),i(z,u)),i(i(i(v,i(i(v,w),w)),z),u)),v6)),v6)),1).
weight(P(i(i(i(x,x),i(i(y,i(i(i(z,i(u,i(i(z,i(u,v)),v))),i(y,w)),w)),v6)),v6)),1).
weight(P(i(i(i(x,x),i(i(y,i(i(i(z,z),i(y,u)),u)),v)),v)),1).
weight(P(i(i(i(x,x),i(i(y,i(z,i(i(y,i(z,u)),u))),v)),v)),1).
weight(P(i(i(i(x,x),i(y,i(i(z,z),u))),i(y,u))),1).
weight(P(i(i(i(x,x),i(y,z)),i(i(i(u,i(i(i(v,v),i(u,w)),w)),y),z))),1).
weight(P(i(i(i(x,x),y),i(i(i(z,z),i(y,u)),u))),1).
weight(P(i(i(i(x,x),y),y)),1).
weight(P(i(i(x,i(i(y,y),z)),i(x,z))),1).
weight(P(i(x,i(i(i(i(i(y,y),i(i(i(i(z,z),i(u,i(i(v,v),w))),i(u,w)),v6)),v6),i(x,v7)),v7))),1).
weight(P(i(x,i(i(i(i(i(y,y),i(i(z,i(u,i(i(z,i(u,v)),v))),w)),w),i(x,v6)),v6))),1).
weight(P(i(x,i(i(i(y,i(z,i(i(y,i(z,u)),u))),i(i(i(i(v,v),i(w,v6)),i(i(x,w),v6)),v7)),v7))),1).
weight(P(i(x,i(i(i(y,i(z,i(i(y,i(z,u)),u))),i(x,v)),v))),1).
weight(P(i(x,i(i(i(y,y),i(i(i(i(z,z),i(u,v)),i(i(x,u),v)),w)),w))),1).
weight(P(i(x,i(i(i(y,y),i(i(i(x,z),z),u)),u))),1).
weight(P(i(x,i(i(i(y,y),i(i(z,i(i(i(i(i(u,u),i(v,w)),i(i(x,v),w)),i(z,v6)),v6)),v7)),v7))),1).
weight(P(i(x,i(i(i(y,y),i(i(z,i(i(x,i(z,u)),u)),v)),v))),1).
weight(P(i(x,i(i(i(y,y),i(x,z)),z))),1).
weight(P(i(x,i(i(x,y),y))),1).
weight(P(i(x,i(y,i(i(i(i(i(z,z),i(x,u)),u),i(y,v)),v)))),1).
weight(P(i(x,i(y,i(i(x,i(y,z)),z)))),1).
weight(P(i(x,x)),1).
%  Following 28 sorted proof steps from temp.halleck.bci.cand46.out1b1.
weight(P(i(i(i(i(i(x,i(i(x,y),y)),z),z),u),u)),1).
weight(P(i(i(i(i(i(x,y),i(z,i(i(z,x),y))),u),i(i(u,v),v))),1).
weight(P(i(i(i(i(x,i(i(x,y),y)),z),z)),1).
weight(P(i(i(i(x,i(y,i(i(x,i(y,z)),z))),i(i(u,i(i(i(v,i(w,i(i(v,i(w,v6)),v6))),i(u,v7)),v7)),v8)),v8)),1).
weight(P(i(i(i(x,i(y,i(i(x,i(y,z)),z))),i(i(u,i(v,i(i(u,i(v,w)),w))),v6)),v6)),1).
weight(P(i(i(i(x,x),i(i(i(i(y,y),i(z,u)),i(i(v,z),u)),w)),i(v,w))),1).
weight(P(i(i(i(x,x),i(i(y,i(i(i(z,i(u,i(i(z,i(u,v)),v))),i(y,w)),w)),v6)),v6)),1).
weight(P(i(i(i(x,x),i(i(y,i(i(i(z,z),i(y,u)),u)),v)),v)),1).
weight(P(i(i(i(x,x),i(i(y,y),i(z,u))),i(i(i(v,v),z),u))),1).
weight(P(i(i(i(x,x),i(i(y,y),i(z,u))),i(v,i(i(v,z),u)))),1).
weight(P(i(i(i(x,x),i(i(y,y),z)),z)),1).
weight(P(i(i(i(x,x),i(y,z)),i(i(i(u,u),y),z))),1).
weight(P(i(i(i(x,x),i(y,z)),i(u,i(i(u,y),z)))),1).
weight(P(i(i(i(x,x),y),y)),1).
weight(P(i(i(x,y),i(i(i(z,i(i(z,x),y)),u),u))),1).
weight(P(i(i(x,y),i(i(i(z,z),x),y))),1).
weight(P(i(i(x,y),i(z,i(i(z,x),y)))),1).
weight(P(i(x,i(i(i(y,i(z,i(i(y,i(z,u)),u))),i(i(i(i(v,v),i(w,v6)),i(i(x,w),v6)),v7)),v7))),1).
weight(P(i(x,i(i(i(y,i(z,i(i(y,i(z,u)),u))),i(x,v)),v))),1).
weight(P(i(x,i(i(i(y,y),i(i(i(i(z,i(u,i(i(z,i(u,v)),v))),i(x,w)),w),v6)),v6))),1).
weight(P(i(x,i(i(i(y,y),i(i(z,i(i(x,i(z,u)),u)),v)),v))),1).
weight(P(i(x,i(i(i(y,y),i(x,z)),z))),1).
weight(P(i(x,i(i(i(y,y),i(z,u)),i(i(x,z),u)))),1).
```

```
weight(P(i(x,i(i(x,i(i(i(y,z),i(u,i(i(u,y),z))),v)),i(i(v,w),w)))),1).
weight(P(i(x,i(i(x,y),i(i(y,z),z)))),1).
weight(P(i(x,i(i(x,y),y))),1).
weight(P(i(x,i(y,i(i(x,i(y,z)),z)))),1).
weight(P(i(x,x)),1).
%  following another, temp.halleck.bci.cand46.out1b2
weight(P(i(i(i(x,i(y,i(i(x,i(y,z)),z))),i(i(i(i(i(i(u,u),i(v,w)),i(i(v6,v),w)),v7),i(v6,v7)),v8)),v8)),1).
%  Following 6 more, temp.halleck.bci.cand46.out1b3.
weight(P(i(x,i(i(i(i(i(y,y),i(z,u)),i(i(x,z),u)),v),v))),1).
weight(P(i(i(i(x,i(y,i(i(x,i(y,z)),z))),i(i(u,i(i(u,v),v)),w)),w)),1).
weight(P(i(i(i(x,i(i(x,i(i(i(i(y,y),i(z,u)),i(i(v,z),u)),w)),i(v,w))),v6),v6)),1).
weight(P(i(i(i(i(i(x,x),i(y,z)),i(i(i(i(u,i(v,i(i(u,i(v,w)),w))),i(i(v6,i(i(v6,v7),v7)),v8)),v8),y),z)),v9),v9)),1).
weight(P(i(i(i(x,i(i(x,i(i(i(i(y,y),i(z,u)),i(i(v,z),u)),w)),i(v,w))),i(i(i(i(v6,v6),i(v7,v8)),
  i(i(v9,v7),v8)),v10)),i(v9,v10))),1).
weight(P(i(i(x,i(i(y,y),z)),i(x,z))),1).
%  Following 5, based on preceding, claim to prove a single from cand46, temp.halleck.bci.cand46.out1m,
%  making cand46 a single axiom, new, 04-16-09.
weight(P(i(x,i(i(i(i(i(i(i(i(y,y),i(z,u)),i(i(v,z),u)),w),i(v,w)),v6),v6),i(x,v7)),v7))),2).
weight(P(i(x,i(i(x,i(i(i(y,z),i(u,i(i(u,y),z))),v)),v))),2).
weight(P(i(i(i(x,x),i(i(i(y,z),u),v)),i(i(i(w,w),i(z,u)),i(y,v)))),2).
weight(P(i(i(i(i(x,y),y),z),i(x,z))),2).
weight(P(i(i(i(i(x,y),z),u),i(i(i(v,v),i(y,z)),i(x,u)))),2).
weight(junk,1000).
end_of_list.

list(usable).
-P(i(x,y)) | -P(x) | P(y).
-P(i(i(a1,i(b,a2)),i(b,i(a1,a2)))) | -P(i(a1,a1)) | -P(i(i(a1,b),i(i(a2,a1),i(a2,b)))) | $ANS(all).  % BCI
end_of_list.

list(sos).
P(i(i(i(i(i(u,u),i(v,w)),i(i(x,v),w)),y),i(x,y))). % BCI-Candidate 46
%  P(i(i(x,y),i(i(i(i(z,x),y),i(i(u,u),v)),i(z,v)))).  % BCI-candidate-22
%  P(i(i(x,y),i(i(i(i(i(z,z),i(y,u)),u),x2),i(x,x2)))).  % BCI candidate 18
%  P(i(i(i(x,x),i(y,i(z,u))),i(i(u,v),i(z,i(y,v))))).  %  a 29th not among the 28 known singles for BCI.
%  P(i(i(x,i(y,z)),i(i(i(u,u),i(v,y)),i(v,i(x,z))))).  % M's BCI #1
end_of_list.

list(passive).
%  Following negs of 72 singles, all of the known, for BCI.
-P(i(i(p,i(q,r)),i(i(i(s,s),i(t,q)),i(t,i(p,r))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(i(s,i(r,t)),i(q,i(s,t))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(r,r),i(i(i(q,s),s),t)),i(p,t)))) | $ANS(TARG2).
-P(i(i(p,q),i(i(q,i(i(r,r),i(s,t))),i(s,i(p,t))))) | $ANS(TARG2).
-P(i(i(i(i(i(p,p),i(q,r)),r),s),i(i(s,t),i(q,t)))) | $ANS(TARG2).
-P(i(i(i(i(p,q),q),r),i(i(i(s,s),i(r,t)),i(p,t)))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(i(i(i(s,s),p),r),t),i(q,t)))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(i(i(s,s),i(r,t)),i(q,i(p,t))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(i(i(q,r),r),s)),i(i(s,t),i(q,t)))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(s,i(i(r,i(s,t)),i(q,t))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(i(i(i(r,s),s),t),i(q,t)))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(i(r,i(s,t)),i(s,i(q,t))))) | $ANS(TARG2).
-P(i(i(p,i(i(q,q),i(r,s))),i(i(s,t),i(r,i(p,t))))) | $ANS(TARG2).
```

-P(i(i(p,i(i(q,q),r)),i(s,i(i(r,i(s,t)),i(p,t))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(r,r),i(q,i(s,t))),i(s,i(p,t))))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(q,i(i(i(s,s),i(r,t)),i(p,t))))) | $ANS(TARG2).
-P(i(p,i(i(i(q,q),i(r,s)),i(i(s,i(p,t)),i(r,t))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(p,r)),i(i(i(s,s),i(r,t)),i(q,t))))) | $ANS(TARG2).
-P(i(i(p,q),i(r,i(i(i(s,s),i(q,i(r,t))),i(p,t))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(i(i(i(s,t),t),q),i(s,r)))) | $ANS(TARG2).
-P(i(p,i(i(i(q,q),i(r,i(p,s))),i(i(t,r),i(t,s))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,i(r,s))),i(i(t,q),i(r,i(t,s))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(r,r),i(i(i(s,p),q),t)),i(s,t)))) | $ANS(TARG2).
-P(i(i(p,i(i(q,q),i(r,s))),i(i(t,p),i(r,i(t,s))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(i(i(i(s,q),r),t),i(s,t)))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(i(s,i(t,q)),i(t,i(s,r))))) | $ANS(TARG2).
-P(i(i(p,i(i(q,q),r)),i(i(r,i(s,t)),i(s,i(p,t))))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(i(i(s,s),i(t,p)),i(q,i(t,r))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(i(i(q,r),i(i(s,q),r)),t)),i(s,t)))) | $ANS(TARG2).
-P(i(i(i(p,p),i(i(i(q,r),r),s)),i(i(t,q),i(t,s)))) | $ANS(TARG2).
-P(i(i(i(p,p),i(i(i(q,r),s),t)),i(i(r,s),i(q,t)))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,i(r,s))),i(i(s,t),i(r,i(q,t))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,i(r,s))),i(i(t,r),i(t,i(q,s))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,i(r,s))),i(r,i(i(s,t),i(q,t))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,i(r,s))),i(r,i(i(t,q),i(t,s))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,i(r,s))),i(t,i(i(t,r),i(q,s))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(q,i(i(s,i(r,t)),i(s,t))))) | $ANS(TARG2).
-P(i(i(i(p,p),q),i(i(i(i(q,i(r,s)),s),t),i(r,t)))) | $ANS(TARG2).
-P(i(i(p,i(i(q,q),i(r,s))),i(i(t,r),i(t,i(p,s))))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(i(s,i(i(t,t),p)),i(q,i(s,r))))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(q,i(i(i(s,s),i(t,p)),i(t,r))))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(s,i(i(i(t,t),i(s,q)),i(p,r))))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(q,i(i(s,i(i(t,t),p)),i(s,r))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(i(i(r,r),i(s,t)),t),p),i(s,q)))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(i(q,r),r),i(i(s,s),t)),i(p,t)))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(i(r,p),q),i(i(s,s),t)),i(r,t)))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(r,r),i(s,i(q,t))),i(p,i(s,t))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(r,r),i(s,i(t,p))),i(t,i(s,q))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(r,i(i(s,s),i(q,t))),i(p,i(r,t))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(r,i(i(s,s),i(t,p))),i(t,i(r,q))))) | $ANS(TARG2).
-P(i(i(p,q),i(p,i(i(i(r,r),i(s,i(q,t))),i(s,t))))) | $ANS(TARG2).
-P(i(i(p,q),i(r,i(i(i(s,s),i(t,i(r,p))),i(t,q))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(i(r,r),i(p,s))),i(i(s,t),i(q,t))))) | $ANS(TARG2).
-P(i(p,i(i(i(q,q),i(r,i(p,s))),i(i(s,t),i(r,t))))) | $ANS(TARG2).
-P(i(p,i(i(i(q,q),i(r,i(s,t))),i(i(p,s),i(r,t))))) | $ANS(TARG2).
-P(i(p,i(i(i(q,q),i(r,s)),i(i(t,i(p,r)),i(t,s))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(i(r,r),i(p,s))),i(i(t,q),i(t,s))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(i(r,r),i(s,t))),i(i(p,s),i(q,t))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(i(r,r),s)),i(i(s,i(p,t)),i(q,t))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(p,r)),i(i(i(s,s),i(t,q)),i(t,r))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(r,s)),i(i(i(t,t),i(p,r)),i(q,s))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(p,r)),i(i(s,i(i(t,t),q)),i(s,r))))) | $ANS(TARG2).
-P(i(p,i(i(p,q),i(i(i(r,r),i(s,i(q,t))),i(s,t))))) | $ANS(TARG2).
-P(i(p,i(i(q,r),i(i(i(s,s),i(r,i(p,t))),i(q,t))))) | $ANS(TARG2).
-P(i(p,i(i(p,q),i(i(r,i(i(s,s),i(q,t))),i(r,t))))) | $ANS(TARG2).
-P(i(p,i(i(q,r),i(i(r,i(i(s,s),i(p,t))),i(q,t))))) | $ANS(TARG2).

```
-P(i(p,i(i(q,r),i(i(s,i(i(t,t),i(p,q))),i(s,r))))) | $ANS(TARG2).
-P(i(i(i(i(i(p,p),i(q,r)),r),s),i(i(t,q),i(t,s)))) | $ANS(TARG2).
-P(i(i(i(i(i(p,p),q),r),s),i(i(q,i(t,r)),i(t,s)))) | $ANS(TARG2).
-P(i(i(i(i(p,i(q,r)),r),s),i(i(i(t,t),p),i(q,s)))) | $ANS(TARG2).
-P(i(i(i(i(p,q),q),r),i(i(i(s,s),i(t,p)),i(t,r)))) | $ANS(TARG2).
-P(i(i(i(i(p,q),r),s),i(i(i(t,t),i(q,r)),i(p,s)))) | $ANS(TARG2).
%  Following 28 are single axioms for BCI.
-P(i(i(a1,i(a2,a3)),i(i(i(a4,a4),i(a5,a2)),i(a5,i(a1,a3))))) | $ANS(TARG).
-P(i(i(i(a1,a1),i(a2,a3)),i(i(a4,i(a3,a5)),i(a2,i(a4,a5))))) | $ANS(TARG).
-P(i(i(a1,a2),i(i(i(a3,a3),i(i(i(a2,a4),a4),a5)),i(a1,a5)))) | $ANS(TARG).
-P(i(i(a1,a2),i(i(a2,i(i(a3,a3),i(a4,a5))),i(a4,i(a1,a5))))) | $ANS(TARG).
-P(i(i(i(i(i(a1,a1),i(a2,a3)),a3),a4),i(i(a4,a5),i(a2,a5)))) | $ANS(TARG).
-P(i(i(i(i(a1,a2),a2),a3),i(i(i(a4,a4),i(a3,a5)),i(a1,a5)))) | $ANS(TARG).
-P(i(i(a1,i(a2,a3)),i(i(i(i(i(a4,a4),a1),a3),a5),i(a2,a5)))) | $ANS(TARG).
-P(i(i(a1,i(a2,a3)),i(i(i(a4,a4),i(a3,a5)),i(a2,i(a1,a5))))) | $ANS(TARG).
-P(i(i(i(a1,a1),i(i(i(a2,a3),a3),a4)),i(i(a4,a5),i(a2,a5)))) | $ANS(TARG).
-P(i(i(i(a1,a1),i(a2,a3)),i(a4,i(i(a3,i(a4,a5)),i(a2,a5))))) | $ANS(TARG).
-P(i(i(i(a1,a1),i(a2,a3)),i(i(i(i(a3,a4),a4),a5),i(a2,a5)))) | $ANS(TARG).
-P(i(i(i(a1,a1),i(a2,a3)),i(i(a3,i(a4,a5)),i(a4,i(a2,a5))))) | $ANS(TARG).
-P(i(i(a1,i(i(a2,a2),i(a3,a4))),i(i(a4,a5),i(a3,i(a1,a5))))) | $ANS(TARG).
-P(i(i(a1,i(i(a2,a2),a3)),i(a4,i(i(a3,i(a4,a5)),i(a1,a5))))) | $ANS(TARG).
-P(i(i(a1,a2),i(i(i(a3,a3),i(a2,i(a4,a5))),i(a4,i(a1,a5))))) | $ANS(TARG).
-P(i(i(a1,i(a2,a3)),i(a2,i(i(i(a4,a4),i(a3,a5)),i(a1,a5))))) | $ANS(TARG).
-P(i(a1,i(i(i(a2,a2),i(a3,a4)),i(i(a4,i(a1,a5)),i(a3,a5))))) | $ANS(TARG).
-P(i(a1,i(i(a2,i(a1,a3)),i(i(i(a4,a4),i(a3,a5)),i(a2,a5))))) | $ANS(TARG).
-P(i(i(a1,a2),i(a3,i(i(i(a4,a4),i(a2,i(a3,a5))),i(a1,a5))))) | $ANS(TARG).
-P(i(i(i(a1,a1),i(a2,a3)),i(i(i(i(a4,a5),a5),a2),i(a4,a3)))) | $ANS(TARG).
-P(i(a1,i(i(i(a2,a2),i(a3,i(a1,a4))),i(i(a5,a3),i(a5,a4))))) | $ANS(TARG).
-P(i(i(i(a1,a1),i(a2,i(a3,a4))),i(i(a5,a2),i(a3,i(a5,a4))))) | $ANS(TARG).
-P(i(i(a1,a2),i(i(i(a3,a3),i(i(i(a4,a1),a2),a5)),i(a4,a5)))) | $ANS(TARG).
-P(i(i(a1,i(i(a2,a2),i(a3,a4))),i(i(a5,a1),i(a3,i(a5,a4))))) | $ANS(TARG).
-P(i(i(i(a1,a1),i(a2,a3)),i(i(i(i(a4,a2),a3),a5),i(a4,a5)))) | $ANS(TARG).
-P(i(i(i(a1,a1),i(a2,a3)),i(i(a4,i(a5,a2)),i(a5,i(a4,a3))))) | $ANS(TARG).
-P(i(i(a1,i(i(a2,a2),a3)),i(i(a3,i(a4,a5)),i(a4,i(a1,a5))))) | $ANS(TARG).
-P(i(i(a1,i(a2,a3)),i(i(i(a4,a4),i(a5,a1)),i(a2,i(a5,a3))))) | $ANS(TARG).
-P(i(a,a)) | $ANS(I).
-P(i(i(a1,i(b,a2)),i(b,i(a1,a2)))) | $ANS(C).  %  C
-P(i(i(a1,b),i(i(a2,a1),i(a2,b)))) | $ANS(B).  % B
end_of_list.

list(demodulators).
%(P(i(i(i(i(i(x,x),i(y,z)),i(y,u)),v),i(i(z,u),v))) = junk).
%  %  (i(i(x,x),y) = junk).
%  %  (i(y,i(x,x)) = junk).
(i(x,junk) = junk).
(i(junk,x) = junk).
(P(junk) = $T).
end_of_list.
```

I now give two proofs that result from using the just-presented input file.

Two Proofs of Interest Obtained from Candidate 46

----- Otter 3.3g-work, Jan 2005 -----
The process was started by wos on octopus.mcs.anl.gov,
Thu Apr 16 19:25:07 2009
The command was "otter". The process ID is 3716.

----> UNIT CONFLICT at 0.80 sec ----> 5028 [binary,5027.1,75.1] $ANS(TARG2).

Length of proof is 29. Level of proof is 19.

---------- PROOF ----------

1 [] -P(i(x,y))| -P(x)|P(y).
3 [] P(i(i(i(i(u,u),i(v,w)),i(i(x,v),w)),y),i(x,y))).
75 [] -P(i(i(i(i(p,q),r),s),i(i(i(t,t),i(q,r)),i(p,s))))|$ANS(TARG2).
110 [hyper,1,3,3] P(i(x,i(y,i(i(x,i(y,z)),z)))).
111 [hyper,1,110,110] P(i(x,i(i(i(y,i(z,i(i(y,i(z,u)),u))),i(x,v)),v))).
114 [hyper,1,111,111] P(i(i(i(x,i(y,i(i(x,i(y,z)),z))),i(i(u,i(i(i(v,i(w,i(i(v,i(w,v6)),v6))),i(u,v7)),v7)),v8)),v8)).
116 [hyper,1,3,111] P(i(x,i(i(i(y,i(z,i(i(y,i(z,u)),u))),i(i(i(i(v,v),i(w,v6)),i(i(x,w),v6)),v7)),v7))).
127 [hyper,1,3,114] P(i(x,i(i(i(y,y),i(i(i(i(z,i(u,i(i(z,i(u,v)),v))),i(x,w)),w),v6)),v6))).
129 [hyper,1,114,116] P(i(i(i(x,x),i(i(y,i(i(i(z,i(u,i(i(z,i(u,v)),v))),i(y,w)),w)),v6)),v6)).
176 [hyper,1,129,127] P(i(x,i(i(i(y,y),i(x,z)),z))).
177 [hyper,1,129,116] P(i(i(i(x,x),i(i(y,i(i(i(z,z),i(y,u)),u)),v)),v)).
217 [hyper,1,176,3] P(i(i(i(x,x),i(i(i(i(i(i(y,y),i(z,u)),i(i(v,z),u)),w),i(v,w)),v6)),v6)).
235 [hyper,1,177,176] P(i(x,x)).
251 [hyper,1,3,235] P(i(x,i(i(i(y,y),i(z,u)),i(i(x,z),u)))).
343 [hyper,1,217,176] P(i(i(i(x,x),i(y,z)),i(i(i(u,u),y),z))).
554 [hyper,1,343,251] P(i(i(i(x,x),i(i(y,y),i(z,u))),i(i(i(v,v),z),u))).
555 [hyper,1,343,217] P(i(i(i(x,x),i(i(i(i(y,y),i(z,u)),i(i(v,z),u)),w)),i(v,w))).
744 [hyper,1,3,554] P(i(i(x,y),i(i(i(z,z),x),y))).
762 [hyper,1,3,555] P(i(i(i(x,x),i(y,z)),i(u,i(i(u,y),z)))).
1156 [hyper,1,762,762] P(i(x,i(i(x,y),i(i(y,z),z)))).
1157 [hyper,1,744,762] P(i(i(i(x,x),i(i(y,y),i(z,u))),i(v,i(i(v,z),u)))).
1185 [hyper,1,3,762] P(i(x,i(y,i(i(y,i(x,z)),z)))).
1620 [hyper,1,3,1157] P(i(i(x,y),i(z,i(i(z,x),y)))).
1864 [hyper,1,1185,1620] P(i(x,i(i(x,i(i(i(y,z),i(u,i(i(u,y),z))),v)),v))).
1865 [hyper,1,1156,1620] P(i(i(i(i(i(x,y),i(z,i(i(z,x),y))),u),i(i(u,v),v))).
2312 [hyper,1,1620,1865] P(i(x,i(i(x,i(i(i(y,z),i(u,i(i(u,y),z))),v)),i(i(v,w),w)))).
3158 [hyper,1,555,2312] P(i(i(x,y),i(i(i(z,i(i(z,x),y)),u),u))).
3607 [hyper,1,3158,3] P(i(i(i(x,i(i(x,i(i(i(i(y,y),i(z,u)),i(i(v,z),u)),w)),i(v,w))),v6),v6)).
3730 [hyper,1,3607,3607] P(i(i(i(x,i(i(x,i(i(i(i(y,y),i(z,u)),i(i(v,z),u)),w)),i(v,w))),
i(i(i(i(v6,v6),i(v7,v8)),i(i(v9,v7),v8)),v10)),i(v9,v10))).
4365 [hyper,1,3730,1864] P(i(i(i(i(x,y),y),z),i(x,z))).
4636 [hyper,1,4365,3730] P(i(i(i(i(x,x),i(i(i(y,z),u),v)),i(i(i(w,w),i(z,u)),i(y,v)))).
5027 [hyper,1,4365,4636] P(i(i(i(i(i(x,y),z),u),i(i(i(v,v),i(y,z)),i(x,u)))).

-----> EMPTY CLAUSE at 21.18 sec ----> 30980 [hyper,2,30613,7407,15755] $ANS(all).

Length of proof is 39. Level of proof is 24.

---------- PROOF ----------

1 [] -P(i(x,y))| -P(x)|P(y).
2 [] -P(i(i(a1,i(b,a2)),i(b,i(a1,a2))))| -P(i(a1,a1))| -P(i(i(a1,b),i(i(a2,a1),i(a2,b))))|$ANS(all).

3 [] P(i(i(i(i(i(u,u),i(v,w)),i(i(x,v),w)),y),i(x,y))).
110 [hyper,1,3,3] P(i(x,i(y,i(i(x,i(y,z)),z)))).
111 [hyper,1,110,110] P(i(x,i(i(i(y,i(z,i(i(y,i(z,u)),u))),i(x,v)),v))).
114 [hyper,1,111,111] P(i(i(i(x,i(y,i(i(x,i(y,z)),z))),i(i(u,i(i(i(v,i(w,i(i(v,i(w,v6)),v6))),v6)),i(u,v7)),v7)),v8)),v8)).
116 [hyper,1,3,111] P(i(x,i(i(i(y,i(z,i(i(y,i(z,u)),u))),i(i(i(i(v,v),i(w,v6)),i(i(x,w),v6)),v7)),v7))).
117 [hyper,1,111,110] P(i(i(i(x,i(y,i(i(x,i(y,z)),z))),i(i(u,i(v,i(i(u,i(v,w)),w))),v6)),v6)).
127 [hyper,1,3,114] P(i(x,i(i(i(y,y),i(i(i(i(z,i(u,i(i(z,i(u,v)),v))),i(x,w)),w),v6)),v6))).
129 [hyper,1,114,116] P(i(i(i(x,x),i(i(y,i(i(i(z,i(u,i(i(z,i(u,v)),v))),i(y,w)),w)),v6)),v6)).
147 [hyper,1,3,117] P(i(x,i(i(i(y,y),i(i(z,i(i(x,i(z,u)),u)),v)),v))).
176 [hyper,1,129,127] P(i(x,i(i(i(y,y),i(x,z)),z))).
177 [hyper,1,129,116] P(i(i(i(x,x),i(i(y,i(i(i(z,z),i(y,u)),u)),v)),v)).
181 [hyper,1,147,147] P(i(i(i(x,x),i(i(y,i(i(i(z,i(i(i(u,u),i(i(v,i(i(z,i(v,w)),w)),v6)),v6)),i(y,v7)),v7)),v8)),v8)).
217 [hyper,1,176,3] P(i(i(i(x,x),i(i(i(i(i(i(y,y),i(z,u)),i(i(v,z),u)),w),i(v,w)),v6)),v6)).
235 [hyper,1,177,176] P(i(x,x)).
251 [hyper,1,3,235] P(i(x,i(i(i(y,y),i(z,u)),i(i(x,z),u)))).
343 [hyper,1,217,176] P(i(i(i(i(x,x),i(y,z)),i(i(i(u,u),y),z))).
554 [hyper,1,343,251] P(i(i(i(i(x,x),i(i(y,y),i(z,u))),i(i(i(v,v),z),u))).
555 [hyper,1,343,217] P(i(i(i(i(x,x),i(i(i(i(y,y),i(z,u)),i(i(v,z),u)),w)),i(v,w))).
744 [hyper,1,3,554] P(i(i(i(x,y),i(i(i(z,z),x),y))).
762 [hyper,1,3,555] P(i(i(i(i(x,x),i(y,z)),i(u,i(i(u,y),z)))).
1156 [hyper,1,762,762] P(i(x,i(i(x,y),i(i(y,z),z)))).
1157 [hyper,1,744,762] P(i(i(i(i(x,x),i(i(y,y),i(z,u))),i(v,i(i(v,z),u)))).
1185 [hyper,1,3,762] P(i(x,i(y,i(i(y,i(x,z)),z)))).
1620 [hyper,1,3,1157] P(i(i(i(x,y),i(z,i(i(z,x),y)))).
1864 [hyper,1,1185,1620] P(i(x,i(i(x,i(i(i(y,z),i(u,i(i(u,y),z))),v)),v))).
1865 [hyper,1,1156,1620] P(i(i(i(i(i(x,y),i(z,i(i(z,x),y))),u),i(i(u,v),v))).
2312 [hyper,1,1620,1865] P(i(x,i(i(x,i(i(i(y,z),i(u,i(i(u,y),z))),v)),i(i(v,w),w)))).
3158 [hyper,1,555,2312] P(i(i(i(x,y),i(i(i(z,i(i(z,x),y)),u),u))).
3607 [hyper,1,3158,3] P(i(i(i(i(x,i(i(x,i(i(i(i(i(y,y),i(z,u)),i(i(v,z),u)),w)),i(v,w))),v6),v6)).
3730 [hyper,1,3607,3607] P(i(i(i(i(x,i(i(x,i(i(i(i(i(y,y),i(z,u)),i(i(v,z),u)),w)),i(v,w))),
i(i(i(i(v6,v6),i(v7,v8)),i(i(v9,v7),v8)),v10)),i(v9,v10))).
4365 [hyper,1,3730,1864] P(i(i(i(i(x,y),y),z),i(x,z))).
4636 [hyper,1,4365,3730] P(i(i(i(i(x,x),i(i(i(y,z),u),v)),i(i(i(w,w),i(z,u)),i(y,v)))).
5027 [hyper,1,4365,4636] P(i(i(i(i(x,y),z),u),i(i(i(v,v),i(y,z)),i(x,u)))).
5083 [hyper,1,3,4636] P(i(i(x,y),i(i(i(z,z),i(y,u)),i(x,u)))).
6049 [hyper,1,5027,4365] P(i(i(i(i(x,x),i(y,z)),i(i(u,y),i(u,z)))).
7407 [hyper,1,181,116] P(i(x,x)).
15755 [hyper,1,4365,6049] P(i(i(x,y),i(i(z,x),i(z,y)))).
16149 [hyper,1,15755,4365] P(i(i(x,i(i(i(y,z),z),u)),i(x,i(y,u)))).
30328 [hyper,1,16149,5083] P(i(i(x,y),i(i(y,z),i(x,z)))).
30613 [hyper,1,16149,30328] P(i(i(x,i(y,z)),i(y,i(x,z)))).

You might have noticed the command set(ancestor_subsume) in the input file. The inclusion of that command invokes a procedure, formulated by McCune, that compares two paths to the same conclusion, preferring the strictly shorter. I use ancestor subsumption heavily when seeking shorter proofs.

If I tell you (as is the case) that I did not find any additional shortest single axioms for *BCI*, you might naturally suspect that the story has come to an end. But, in fact, more piquancy, strangeness, and charm awaits you—here, and in the final section.

Indeed, in the context of candidate 42, which appeared to be unpromising, M. Stickel proved otherwise (with his program SNARK). He showed with his program that that formula is in fact also a single axiom. The proof he found is astounding in the following ways. His proof includes a formula, the following, that rests on thirty-four distinct variables and is indeed long.

189 [hyper,1,176,175] P(i(i(i(x,x),i(i(y,i(i(z,u),i(i(i(v,v),i(w,i(y,z))),i(w,u)))),i(i(v6,i(i(v7,v8),

i(i(i(v9,v9),i(v10,i(v6,v7))),i(v10,v8)))),i(i(v11,v11),i(i(v12,v12),i(i(v13,i(i(v14,v15),
i(i(i(v16,v16),i(v17,i(v13,v14))),i(v17,v15)))),i(v18,v19)))))))),i(i(i(v20,v20),i(v21,
i(i(v22,i(i(v23,v24),i(i(i(v25,v25),i(v26,i(v22,v23))),i(v26,v24)))),i(i(v27,v27),
i(i(v28,v28),i(i(v29,i(i(v30,v31),i(i(i(v32,v32),i(v33,i(v29,v30))),i(v33,v31)))),v18))))))),i(v21,v19)))).

Stickel's breakthrough in regard to candidate 42 was that of deducing eventually a formula in which no alphabetic variant of $i(x,x)$ occurs. He had learned of the open question from the cited AAR newsletter.. Stickel answered the question and communicated to me by e-mail his answer one day after the column was made available.

4. Epilogue, Overview, and Odds and Ends

This notebook offers evidence that various methodologies used in proof shortening can successfully be used in finding a first proof, in, for example, settling a conjecture or in answering an open question. Indeed, I found the first proof ever known that each of candidates 23, 51, and 46 turns out to be a (shortest) single axiom for the *BCI* logic. What I, and I believe you will, find additionally interesting is the fact that the first proof for each of these three formulas, when pairwise compared, do not share many deduced steps in common. That fact suggests that the methodologies offered here are indeed quite general. Further, as indicated throughout, you need not be an expert in the area under investigation, and you can, for example, borrow proof steps from closely or distantly related areas to assist the program in use. Iteration is often a key move to make, adjoining or replacing items used to direct or to restrict the reasoning.

The following two approaches, not covered in detail here, that I have frequently used in proof shortening might also prove useful in proof finding and in attacking open questions. In the first, consider the case in which the target consists of, say, three items, as is the case in seeking to deduce the 3-axiom system of the *BCI* logic featured here. If, in some experiment, your program deduces *B*, for example, then the proof steps of that proof can be adjoined, in the next experiment, to list(sos). The motivation rests with the fact that they proved to be powerful, and their use should be encouraged and might enable the program to deduce the remaining members of the target conjunction. This action is what I take when applying the cramming strategy. You can use a level-saturation (breadth-first) search, or a complexity-preference search, or, with McCune's assign(pick_given_ratio), combine the two. ###LARRY - by latter do you mean in the combined? In the latter, with OTTER, you had best include the command set(input_sos_first). The command has the program focus on all items, for inference-rule initiation, in the initial set of support before focusing on deduced items. You would wish the program to do so especially when you have added extra items in list(sos). (As an aside, when you are seeking a first proof or a shorter proof of, say, a conjunction, you should not be surprised when proofs of individual members get shorter but the proof of the entire conjunction gets longer, as the following example taken from a study of the *BCK* logic illustrates.

----> UNIT CONFLICT at 1.15 sec ----> 3150 [binary,3149.1,4.1] $ANS(C).
Length of proof is 32. Level of proof is 24.
----> UNIT CONFLICT at 1.15 sec ----> 3154 [binary,3153.1,4.1] $ANS(C).
Length of proof is 31. Level of proof is 24.
----> UNIT CONFLICT at 1.15 sec ----> 3162 [binary,3161.1,4.1] $ANS(C).
Length of proof is 30. Level of proof is 24.
-----> EMPTY CLAUSE at 1.86 sec ----> 4690 [hyper,2,3161,484,3255] $ANS(all).
Length of proof is 31. Level of proof is 25.
-----> EMPTY CLAUSE at 1737.84 sec ----> 450022 [hyper,2,3161,449860,3255] $ANS(all).
Length of proof is 39. Level of proof is 25.
----> UNIT CONFLICT at 2734.97 sec ----> 661476 [binary,661475.1,4.1] $ANS(C).
Length of proof is 27. Level of proof is 17.
----> UNIT CONFLICT at 2735.03 sec ----> 661495 [binary,661494.1,4.1] $ANS(C).
Length of proof is 26. Level of proof is 17.
----> UNIT CONFLICT at 2735.19 sec ----> 661620 [binary,661619.1,4.1] $ANS(C).
Length of proof is 25. Level of proof is 17.
-----> EMPTY CLAUSE at 2827.42 sec ----> 662321 [hyper,2,661619,449860,3255] $ANS(all).
Length of proof is 54. Level of proof is 25.

-----> EMPTY CLAUSE at 3325.38 sec ----> 670260 [hyper,2,661619,449860,662612] $ANS(all).
Length of proof is 26. Level of proof is 18.
----> UNIT CONFLICT at 8490.77 sec ----> 770044 [binary,770043.1,4.1] $ANS(C).
Length of proof is 24. Level of proof is 17.
-----> EMPTY CLAUSE at 8551.06 sec ----> 770391 [hyper,2,770043,449860,662612] $ANS(all).
Length of proof is 29. Level of proof is 18.
-----> EMPTY CLAUSE at 9375.92 sec ----> 779550 [hyper,2,770043,449860,770719] $ANS(all).
Length of proof is 26. Level of proof is 18.

In other words, sometimes progress measured in one parameter is in fact not progress measured in terms of the most pertinent parameter.)

The second bit of methodology concerns the use of demodulation, alternately, weighting (from Overbeek). The basic idea is to purge immediately upon generation any unwanted item. In the context of first-proof finding, you might—especially with expertise in the field of study—conjecture that the retention of some particular formula or equation will result in the program wandering far afield. Of course, the two cited methodologies can be combined and can also be merged with those discussed earlier here.

Obviously, I am recommending use, as an assistant, of a powerful automated reasoning program. After all, the space of conclusions that can be examined is gigantic. Sometimes, the program ventures deep into that space before the goal is reached. A fine example of this phenomenon, of large numbers, is provided by the following proof, obtained when candidate 48 (for *BCI*) was studied.

P(i(i(i(i(i(u,u),i(v,w)),x),y),i(i(w,x),i(v,y)))). % BCI-Candidate 48

A Proof Exhibiting Large Numbers, Focusing on Candidate 48

----- Otter 3.3g-work, Jan 2005 -----
The process was started by wos on elephant.mcs.anl.gov,
Fri Apr 24 12:12:24 2009
The command was "otter". The process ID is 17948.

-----> EMPTY CLAUSE at 190706.49 sec ----> 2142345 [hyper,2,2094994,2061665,2141277] $ANS(all).

Length of proof is 20. Level of proof is 14.

---------- PROOF ----------

1 [] -P(i(x,y))| -P(x)|P(y).
2 [] -P(i(i(a1,i(b,a2)),i(b,i(a1,a2))))| -P(i(a1,a1))| -P(i(i(a1,b),i(i(a2,a1),i(a2,b))))|$ANS(all).
3 [] P(i(i(i(i(i(u,u),i(v,w)),x),y),i(i(w,x),i(v,y)))).
111 [hyper,1,3,3] P(i(i(x,y),i(z,i(i(u,i(z,x)),i(u,y))))).
113 [hyper,1,3,111] P(i(i(x,y),i(z,i(u,i(i(v,i(u,i(i(w,w),i(z,x)))),i(v,y)))))).
119 [hyper,1,113,3] P(i(x,i(y,i(i(z,i(y,i(i(u,u),i(x,i(i(i(i(v,v),i(w,v6)),v7),v8)
))))),i(z,i(i(v6,v7),i(w,v8)))))).
124 [hyper,1,119,111] P(i(x,i(i(y,i(x,i(i(z,z),i(i(i(u,v),i(w,i(i(v6,i(w,u)),i(v6,v)))),
i(i(i(i(v7,v7),i(v8,v9)),v10),v11))))),i(y,i(i(v9,v10),i(v8,v11)))))).
130 [hyper,1,124,3] P(i(i(x,i(i(i(i(i(y,y),i(z,u)),v),w),i(i(u,v),i(z,w))),i(i(v6,v6),i(i(i(v7,v8),
i(v9,i(i(v10,i(v9,v7)),i(v10,v8)))),i(i(i(i(v11,v11),i(v12,v13)),v14),v15))))),i(x,i(i(v13,v14),i(v12,v15))))).
133 [hyper,1,130,113] P(i(i(x,y),i(i(i(i(z,z),i(u,v)),v),x),i(u,y)))).
138 [hyper,1,124,133] P(i(i(x,i(i(i(y,z),i(i(i(i(i(u,u),i(v,w)),w),y),i(v,z))),i(i(v6,v6),
i(i(i(v7,v8),i(v9,i(i(v10,i(v9,v7)),i(v10,v8)))),i(i(i(i(v11,v11),i(v12,v13)),v14),v15))))),
i(x,i(i(v13,v14),i(v12,v15))))).
147 [hyper,1,133,3] P(i(i(i(i(i(x,x),i(y,z)),z),i(i(i(i(u,u),i(v,w)),v6),v7)),i(y,i(i(w,v6),i(v,v7))))).
160 [hyper,1,138,133] P(i(i(i(i(i(i(i(x,x),i(y,z)),z),i(i(u,u),i(i(v,v),w))),i(y,w)),i(i(i(v6,v7),

i(v8,i(i(v9,i(v8,v6)),i(v9,v7)))),i(i(i(i(v10,v10),i(v11,v12)),v13),v14))),i(i(v12,v13),i(v11,v14)))).
39959 [hyper,1,160,147] P(i(i(x,i(i(y,y),i(i(z,z),u))),i(i(v,x),i(v,u)))).
2059292 [hyper,1,160,39959] P(i(i(x,i(i(y,y),z)),i(x,z))).
2059902 [hyper,1,39959,111] P(i(i(x,i(y,z)),i(x,i(i(i(u,u),y),z)))).
2060871 [hyper,1,2059292,2059292] P(i(i(i(x,x),i(i(y,y),z)),z)).
2061642 [hyper,1,2059292,133] P(i(i(i(i(i(x,x),i(y,z)),z),u),i(y,u))).
2061665 [hyper,1,2060871,2060871] P(i(x,x)).
2076657 [hyper,1,2059292,2059902] P(i(i(i(x,x),i(y,z)),i(i(i(u,u),y),z))).
2091499 [hyper,1,2061642,3] P(i(x,i(i(y,i(x,z)),i(y,z)))).
2092174 [hyper,1,2091499,2091499] P(i(i(x,i(i(y,i(i(z,i(y,u)),i(z,u))),v)),i(x,v))).
2094994 [hyper,1,2092174,2091499] P(i(i(x,i(y,z)),i(y,i(x,z)))).
2141277 [hyper,1,147,2076657] P(i(i(x,y),i(i(z,x),i(z,y)))).

The question of whether candidate 48 is a single axiom for *BCI* was open until Stickel and his program studied it. He, clearly, answered it in the affirmative. Without his success and subsequent proof, the preceding would almost certainly never have been found by me. His original proof relies on fifty-five distinct variables, containing a formula of length 220, the following.

611 [hyper,1,468,319] P(i(i(i(i(x,y),i(z,i(u,i(v,i(i(w,i(v,i(i(v6,v6),i(u,i(i(v7,v7),i(z,x)))))),
i(w,y)))))),i(i(i(v8,v9),i(v10,i(v11,i(v12,i(i(v13,i(v12,i(i(v14,v14),i(v11,i(i(v15,v15),
i(v10,v8)))))),i(v13,v9)))))),i(i(v16,v16),i(i(i(v17,v18),i(v19,i(v20,i(i(v21,i(v20,i(i(v22,v22),
i(v19,v17)))),i(v21,v18))))),i(i(i(i(v23,v23),i(i(i(v24,v25),i(v26,i(v27,i(i(v28,i(v27,
i(i(v29,v29),i(v26,v24)))),i(v28,v25))))),v30)),v30),i(i(i(i(v31,v31),i(i(i(v32,v33),i(v34,
i(v35,i(i(v36,i(v35,i(i(v37,v37),i(v34,v32)))),i(v36,v33))))),i(i(v38,v38),i(i(i(v39,v40),
i(v41,i(v42,i(i(v43,i(v42,i(i(v44,v44),i(v41,v39)))),i(v43,v40))))),v45)))),v45),i(i(i(v46,v47),
i(v48,i(v49,i(v50,i(i(v51,i(v50,i(i(v52,v52),i(v49,i(i(v53,v53),i(v48,v46)))))),
i(v51,v47)))))),v54)))))),v54)).

I think it safe to conjecture that, unaided, no researcher would have discovered the Stickel proof, and, more important, candidate 48 might still be a puzzle in the context of being a single axiom for *BCI*. Stickel also answered in the affirmative a comparable question regarding the following formula, candidate 19.

P(i(i(u,v),i(i(i(i(w,w),i(x,u)),v),y),i(x,y)))). % BCI-Candidate 19

Is your excitement and curiosity growing in the context of what can be done with automated reasoning? I am, I must note, impressed with the ability to cope with such monstrous formulas.

Although the majority of this notebook focuses on finding a first proof, any first proof with no conditions to be satisfied, sometimes the goal is that of finding a first proof of some given type. For example, you might be seeking a proof that avoids the use of some unwanted lemma, or a proof that avoids items more complex than a given amount, or a proof none of whose deduced steps relies on more than k distinct variables, where you have chosen k. Each of these types of first proof can be found with OTTER. Indeed, I have sought first proofs of each of the three given types in my experiments with various areas of logic and algebra. To avoid the use of one or more specified lemmas, you can use demodulation (by rewriting such to junk) or by placing the corresponding item in weight_list(pick_and_purge) with an assigned weight larger than the value assigned to max_weight. If, on the other hand, you wish to place an upper bound on the complexity of any deduced step occurring in a proof, you assign the corresponding value to max_weight. In such an event, of course, you must take into account any weight templates that interfere with the program computing complexity simply in terms of symbol count. As for a concern for distinct variables, the max_distinct_vars item in input files given here is what is used. Indeed, if the value assigned to max_distinct_vars if j, then OTTER will discard any new conclusion that relies on strictly more than j distinct variables. When the focus is on that aspect, sometimes the program completes and offers to you a proof whose formula complexity is markedly less than that in hand.

The following proof might increase your enjoyment, interest, or zeal regarding the use of an automated reasoning program. The proof focuses on the use of candidate 48 as the sole hypothesis. The general goal, concerned with a type of simplification, was in part motivated by various e-mail I have received from Ulrich, and in part by the desire to find out the truth. In particular, Ulrich prefers proofs in which less

complexity, in comparison to more, is present, even if the simpler proof is longer than the more complex. He typically focuses on the length in symbol count of the longest deduced step. My experiment, somewhat related to his preference, was concerned with finding a first proof in which no deduced step relied upon strictly more than sixteen distinct variables. I did not explicitly have in mind the possible side effect on formula complexity. Of course, none of my studies of candidate 48 would have been possible were it not for Stickel's having supplied a proof that candidate 48 was so powerful that its use permits the deduction of the conjunction of *B*, *C*, and *I*. I now present a proof that, to me, was far more than a surprise, one that Ulrich would enjoy in part because of its length and in part because the conclusion is the deduction of a formula that was known to be a single axiom. I follow the proof with the appropriate input file.

A Most Unusual Proof Based on the Use of Candidate 48

```
----- Otter 3.3g-work, Jan 2005 -----
The process was started by wos on octopus.mcs.anl.gov,
Tue Apr 21 20:41:24 2009
The command was "otter".  The process ID is 25432.

----> UNIT CONFLICT at 490.95 sec ----> 69367 [binary,69366.1,99.1] $ANS(TARG2).

Length of proof is 6.  Level of proof is 6.

---------- PROOF ----------

1 [] -P(i(x,y))| -P(x)|P(y).
3 [] P(i(i(i(i(i(u,u),i(v,w)),x),y),i(i(w,x),i(v,y)))).
99 [] -P(i(i(p,q),i(i(i(i(i(r,r),i(s,t)),t),p),i(s,q))))|$ANS(TARG2).
210 [hyper,1,3,3] P(i(i(x,y),i(z,i(i(u,i(z,x)),i(u,y))))).
213 [hyper,1,3,210] P(i(i(x,y),i(z,i(u,i(i(v,i(u,i(i(w,w),i(z,x)))),i(v,y)))))).
228 [hyper,1,213,3] P(i(x,i(y,i(i(z,i(y,i(i(u,u),i(x,i(i(i(i(v,v),i(w,v6)),v7),v8))))),
  i(z,i(i(v6,v7),i(w,v8)))))).
434 [hyper,1,228,210] P(i(x,i(i(y,i(x,i(i(z,z),i(i(i(u,v),i(w,i(i(v6,i(w,u)),i(v6,v))))),
  i(i(i(i(v7,v7),i(v8,v9)),v10),v11))))),i(y,i(i(v9,v10),i(v8,v11)))))).
3046 [hyper,1,434,3] P(i(i(x,i(i(i(i(i(i(y,y),i(z,u)),v),w),i(i(u,v),i(z,w))),i(i(v6,v6),
  i(i(i(v7,v8),i(v9,i(i(v10,i(v9,v7)),i(v10,v8)))),i(i(i(i(v11,v11),i(v12,v13)),v14),v15))))),
  i(x,i(i(v13,v14),i(v12,v15))))).
69366 [hyper,1,3046,213] P(i(i(x,y),i(i(i(i(i(z,z),i(u,v)),v),x),i(u,y)))).
```

An Input File Concerned with a Most Unusual Proof

```
set(hyper_res).
assign(max_weight,80).
%  assign(change_limit_after,800).
%  assign(new_max_weight,22).
assign(max_proofs,-1).
clear(print_kept).
set(ancestor_subsume).
set(back_sub).
% clear(for_sub).
clear(print_back_sub).
clear(print_kept).
clear(print_new_demod).
clear(print_back_demod).
```

```
clear(print_back_sub).
assign(max_distinct_vars,16).
%  assign(pick_given_ratio,2).
assign(max_mem,750000).
%  assign(max_seconds,2).
assign(report,5400).
set(order_history).
set(input_sos_first).
set(sos_queue).
assign(bsub_hint_wt,1).
set(keep_hint_subsumers).

weight_list(pick_and_purge).
weight(P(i(i(x,y),i(i(z,x),i(z,y)))),-8).  % B
weight(P(i(i(x,i(y,z)),i(y,i(x,z)))),-8).  % C
weight(P(i(x,x)),-8).  % I
%  Following 42 apparently prove candidate 18 a new single axiom, from Halleck.
weight(P(i(i(x,y),i(i(i(i(i(z,z),i(y,u)),u),x2),i(x,x2)))),-4).  % BCI candidate 18
weight(P(i(i(i(i(i(x,x),i(i(i(i(i(i(y,y),i(z,u)),u),x1),i(u,x1)),v)),v),w),i(i(u,z),w))),-4).
weight(P(i(i(x,i(y,z)),i(i(u,y),i(u,i(x,z))))),-4).
weight(P(i(i(i(i(i(x,x),i(i(i(y,z),u),x2)),x2),u),i(i(i(i(i(v,v),i(i(i(i(i(w,w),i(z,x)),x),y),i(y,y)),z)),z),u),u))),-4).
weight(P(i(i(i(i(i(x,x),i(i(i(i(i(i(y,y),i(z,u)),u),x2),i(u,x2)),v)),v),w),i(i(x,y),i(x,i(i(i(z,z),
  i(i(i(u,z),w),i(y,x1))),x1))))),-4).
weight(P(i(i(x,y),i(x,i(i(i(z,z),i(i(i(u,i(x2,u)),i(i(v,x2),i(v,i(u,u)))),i(y,w))),w)))),-4).
weight(P(i(i(x,i(i(i(i(y,y),i(z,u)),u),x2)),i(x,i(i(u,z),i(u,x2))))),-4).
weight(P(i(i(x,i(y,z)),i(i(u,x2),i(u,i(i(i(u,u),i(x2,y)),i(x,z)))))),-4).
weight(P(i(i(x,y),i(x,i(i(i(z,z),i(y,i(i(i(i(u,u),i(x2,u)),u),v))),i(i(w,x2),i(w,v)))))),-4).
weight(P(i(i(x,i(y,z)),i(x,i(i(u,i(z,x2)),i(y,i(u,x2)))))),-4).
weight(P(i(i(x,i(y,z)),i(i(u,i(i(i(x2,i(z,u)),i(y,i(x2,u))),v)),i(x,i(u,v))))),-4).
weight(P(i(i(x,i(y,i(i(i(z,i(u,x2)),i(u,i(z,x2))),v))),i(x,i(i(w,i(u,u)),i(w,i(y,v)))))),-4).
weight(P(i(i(x,y),i(i(z,i(y,i(i(u,u),i(x2,u)))),i(z,i(x,i(i(v,x2),i(v,u)))))),-4).
weight(P(i(i(x,i(y,z)),i(i(u,i(i(x2,i(x,z)),u)),i(i(x2,y),i(u,u))))),-4).
weight(P(i(i(x,i(i(y,i(i(z,i(u,x2)),i(u,i(z,x2)))),v)),i(i(y,i(u,u)),i(x,v)))),-4).
weight(P(i(i(x,i(i(y,y),z)),i(i(u,i(x2,i(z,u))),i(x,i(u,i(i(v,x2),i(v,u)))))),-4).
weight(P(i(i(x,i(y,i(i(z,i(i(i(u,u),i(x2,u)),u)),v))),i(i(z,x2),i(x,i(i(w,y),i(w,v)))))),-4).
weight(P(i(i(i(x,x),i(y,z)),i(i(u,y),i(i(x2,u),i(x2,i(i(z,u),u)))))),-4).
weight(P(i(i(x,i(i(i(i(y,y),i(z,u)),u),x2)),i(i(u,x),i(u,i(i(i(z,x2),v),v))))),-4).
weight(P(i(i(x,i(y,i(z,u))),i(x,i(i(i(z,i(y,u)),x2),x2)))),-4).
weight(P(i(i(x,i(y,z)),i(i(i(u,i(i(u,y),i(x,z))),x2),x2))),-4).
weight(P(i(i(x,i(y,z)),i(i(u,i(i(x2,x2),y)),i(u,i(x,z))))),-4).
weight(P(i(i(i(i(i(x,x),i(i(i(y,z),i(y,i(u,x2))),u)),u),v),i(i(u,i(z,x2)),v))),-4).
weight(P(i(i(x,i(y,i(z,u))),i(i(i(i(i(z,i(y,u)),x2),i(x,x2)),u),u))),-4).
weight(P(i(i(i(i(i(x,i(i(x,y),i(z,u))),x2),i(i(z,i(y,u)),x2)),u),u)),-4).
weight(P(i(i(x,i(y,z)),i(i(u,i(i(y,i(x,z)),x2)),i(u,x2)))),-4).
weight(P(i(i(x,i(i(i(i(y,z),i(i(u,i(z,x2)),i(y,i(u,x2)))),u)),i(x,u))),-4).
weight(P(i(i(x,i(y,z)),i(i(u,x),i(u,i(y,z))))),-4).
weight(P(i(i(x,i(y,z)),i(i(i(u,i(i(u,x),i(y,z))),x2),x2))),-4).
weight(P(i(i(x,i(y,z)),i(i(u,i(i(x2,x2),x)),i(u,i(y,z))))),-4).
weight(P(i(i(i(x,i(i(y,z),z)),i(u,x2)),i(i(x,y),i(u,x2)))),-4).
weight(P(i(i(x,i(y,z)),i(i(i(u,u),i(x2,y)),i(x2,i(x,z))))),-4).
weight(P(i(i(x,i(i(y,i(z,u)),x2)),i(i(z,i(y,u)),i(x,x2)))),-4).
weight(P(i(i(i(x,x),i(y,z)),i(i(u,i(z,x2)),i(y,i(u,x2))))),-4).
weight(P(i(i(x,i(i(i(i(y,i(z,u)),x2),x2),u)),i(i(z,i(y,u)),i(x,u)))),-4).
```

```
weight(P(i(i(x,i(i(y,y),z)),i(i(u,x),i(u,z)))),-4).
weight(P(i(i(x,y),i(i(z,x),i(z,y)))),-4).  % B
weight(P(i(i(x,i(y,z)),i(x,i(y,z)))),-4).
weight(P(i(i(x,i(i(y,y),i(z,u))),i(x,i(z,u)))),-4).
weight(P(i(i(x,i(y,z)),i(y,i(x,z)))),-4).  % C
weight(P(i(i(x,y),i(x,y))),-4).
weight(P(i(i(i(x,x),i(y,z)),i(y,z))),-4).
weight(P(i(x,x)),-4).
%  Following 23/16 prove join of BCI from the 29th odd one, temp.bci.halleck.out2e.
weight(P(i(i(i(i(x,y),z),u),i(x,i(i(y,z),u)))),-4).
weight(P(i(i(i(i(x,y),z),u),i(i(y,z),i(x,u)))),-4).
weight(P(i(i(x,y),i(i(z,u),i(z,i(i(u,x),y))))),-4).
weight(P(i(i(i(i(x,y),y),z),i(u,i(i(u,x),z)))),-4).
weight(P(i(x,i(i(x,i(y,z)),i(u,i(i(u,y),z))))),-4).
weight(P(i(x,i(i(x,i(y,z)),i(y,i(i(z,u),u))))),-4).
weight(P(i(i(i(i(x,y),y),z),i(u,i(i(i(v,v),i(u,x)),z)))),-4).
weight(P(i(i(i(i(x,y),y),z),i(i(i(u,u),i(v,x)),i(v,z)))),-4).
weight(P(i(i(i(x,x),i(y,i(z,u))),i(y,i(i(u,v),i(z,v))))),-4).
weight(P(i(i(i(x,x),i(y,z)),i(i(i(i(z,u),u),v),i(y,v)))),-4).
weight(P(i(i(i(x,x),i(y,z)),i(i(i(i(u,y),z),v),i(u,v)))),-4).
weight(P(i(i(x,y),i(i(i(i(y,z),z),u),i(x,u)))),-4).
weight(P(i(i(i(i(x,y),y),z),i(i(z,u),i(x,u)))),-4).
weight(P(i(i(i(i(x,i(i(i(y,z),z),u)),i(y,u)),v),i(x,v))),-4).
weight(P(i(i(i(i(i(x,x),i(y,z)),i(y,u)),v),i(i(z,u),v))),-4).
weight(P(i(i(x,i(i(i(i(i(y,z),z),u),u),v)),i(x,i(y,v)))),-4).
weight(P(i(i(x,y),i(i(z,x),i(z,y)))),-4).
weight(P(i(i(x,i(y,z)),i(y,i(x,z)))),-4).
weight(P(i(i(i(x,x),i(y,i(z,u))),i(z,i(y,u)))),-4).
weight(P(i(x,i(i(i(y,y),i(z,u)),i(i(x,z),u)))),-4).
weight(P(i(i(i(x,x),y),i(i(i(z,z),i(y,u)),u))),-4).
weight(P(i(i(i(x,x),i(i(y,i(i(y,z),z)),u)),u)),-4).
weight(P(i(x,x)),-4).
%  Following 31 include Meredith's single for BCK, and his 30-step proof.
weight(P(i(i(i(u,v),w),i(i(x,i(w,y)),i(v,i(x,y))))),2).  %  Meredith's single axiom for BCK
weight(P(i(i(u,i(i(i(v,i(w,x)),i(y,i(v,x))),z)),i(w,i(u,z)))),2).
weight(P(i(u,i(i(i(v,w),x),i(w,i(i(y,i(u,z)),i(y,z)))))),2).
weight(P(i(u,i(v,i(i(u,w),i(i(x,i(v,y)),i(x,y)))))),2).
weight(P(i(u,i(i(i(i(i(v,w),x),i(i(y,i(x,z)),i(w,i(y,z)))),v6),i(i(v7,i(u,v8)),i(v7,v8))))),2).
weight(P(i(i(u,i(v,w)),i(x,i(i(y,i(x,z)),i(y,z))))),2).
weight(P(i(u,i(i(v,i(u,w)),i(v,w)))),2).
weight(P(i(i(u,i(i(v,i(i(w,i(v,x)),i(w,x))),y)),i(u,y))),2).
weight(P(i(i(u,i(v,w)),i(v,i(u,w)))),2).  %  C
weight(P(i(i(i(u,i(i(v,i(u,w)),i(v,w))),i(i(x,i(i(y,i(x,z)),i(y,z))),v6)),v6)),2).
weight(P(i(i(u,i(v,w)),i(i(i(x,i(i(y,i(x,z)),i(y,z))),v),i(u,w)))),2).
weight(P(i(i(u,i(i(i(i(v,i(i(w,i(v,x)),i(w,x))),y),i(z,v6)),v7)),i(i(y,v6),i(u,v7)))),2).
weight(P(i(i(u,v),i(w,i(i(i(x,i(i(y,i(x,z)),i(y,z))),u),v)))),2).
weight(P(i(i(u,v),i(i(i(w,i(i(x,i(w,y)),i(x,y))),u),v))),2).
weight(P(i(i(i(u,i(i(v,i(u,w)),i(v,w))),i(x,y)),i(i(i(z,i(i(v6,i(z,v7)),i(v6,v7))),x),y))),2).
weight(P(i(i(u,i(i(i(i(v,i(i(w,i(v,x)),i(w,x))),y),z),v6)),i(i(y,z),i(u,v6)))),2).
weight(P(i(i(u,i(i(v,i(w,x)),y)),i(i(v,x),i(u,y)))),2).
weight(P(i(i(u,v),i(w,i(u,v)))),2).
weight(P(i(i(u,i(i(v,i(w,x)),y)),i(x,i(u,y)))),2).
weight(P(i(u,i(v,i(w,u))))),2).  %  K
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weight(P(i(u,i(v,u))),2). % K
weight(P(i(u,i(i(i(v,w),x),i(i(y,i(x,z)),i(w,i(y,z))))),2).
weight(P(i(i(u,v),i(w,i(i(x,i(v,y)),i(u,i(x,y))))),2).
weight(P(i(i(u,v),i(i(w,i(v,x)),i(u,i(w,x))))),2).
weight(P(i(i(u,i(v,w)),i(i(x,v),i(x,i(u,w))))),2).
weight(P(i(i(u,v),i(u,v))),2).
weight(P(i(i(u,v),i(u,i(i(v,w),w)))),2).
weight(P(i(i(u,v),i(i(i(u,i(i(v,w),w)),x),x))),2).
weight(P(i(i(i(u,i(i(v,w),w)),x),i(i(u,v),x))),2).  %  B'
weight(P(i(i(u,v),i(i(v,w),i(u,w)))),2).  %  B'
weight(P(i(i(u,v),i(i(w,u),i(w,v)))),2).  %  B
end_of_list.

list(usable).
-P(i(x,y)) | -P(x) | P(y).
-P(i(i(a1,i(b,a2)),i(b,i(a1,a2)))) | -P(i(a1,a1)) | -P(i(i(a1,b),i(i(a2,a1),i(a2,b)))) | $ANS(all).  % BCI
end_of_list.

list(sos).
P(i(i(i(i(i(u,u),i(v,w)),x),y),i(i(w,x),i(v,y)))). % BCI-Candidate 48
%  Following 13 candidates for being a single axiom for BCI, not yet classified, 04-08-09.
%  P(i(i(u,i(i(v,v),w)),i(i(i(i(x,y),y),u),i(x,w)))). % BCI-Candidate 13
%  P(i(i(u,v),i(i(i(i(i(w,w),i(x,u)),v),y),i(x,y)))). % BCI-Candidate 19
%  P(i(i(u,v),i(i(i(i(w,x),x),i(i(y,y),u)),i(w,v)))). % BCI-Candidate 23
%  P(i(u,i(i(v,w),i(i(i(x,x),i(y,i(u,v))),i(y,w))))). % BCI-Candidate 42
%  P(i(i(i(i(i(u,u),i(v,w)),i(i(x,v),w)),y),i(x,y))). % BCI-Candidate 46
%  P(i(i(i(i(i(u,u),i(v,w)),x),y),i(i(w,x),i(v,y)))). % BCI-Candidate 48
%  P(i(i(i(i(u,v),i(i(w,u),v)),i(i(x,x),y)),i(w,y))). % BCI-Candidate 51
%  P(i(i(i(i(u,v),v),i(i(w,w),x)),i(i(x,y),i(u,y)))). % BCI-Candidate 52
%  P(i(i(i(i(u,v),v),i(i(w,w),x)),i(i(y,u),i(y,x)))). % BCI-Candidate 53
%  P(i(i(i(i(u,v),w),i(i(x,x),y)),i(i(v,w),i(u,y)))). % BCI-Candidate 54
%  P(i(i(i(i(u,v),v),w),i(i(w,i(i(x,x),y)),i(u,y)))). % BCI-Candidate 57
%  P(i(i(x,y),i(i(i(i(i(z,z),i(y,u)),u),x2),i(x,x2)))).  % BCI candidate 18
%  P(i(i(i(x,x),i(y,i(z,u))),i(i(u,v),i(z,i(y,v))))).  %  a 29th not among the 28 known singles for BCI.
%  P(i(i(x,i(y,z)),i(i(i(u,u),i(v,y)),i(v,i(x,z))))).  % M's BCI #1
end_of_list.

list(passive).
%  Following 52 negs of a 52-step proof from cand48 of join BCI, temp.halleck.bci.cand48.out1b8.
-P(i(i(a,b),i(c,i(i(d,i(c,a)),i(d,b))))) | $ANS(TARG6).
-P(i(i(a,b),i(c,i(d,i(i(e,i(d,i(i(f,f),i(c,a)))),i(e,b)))))) | $ANS(TARG6).
-P(i(a,i(i(b,i(a,i(i(i(i(c,c),i(d,e)),f),e6))),i(b,i(i(e,f),i(d,e6)))))) | $ANS(TARG6).
-P(i(a,i(b,i(i(c,i(b,i(i(d,d),i(a,i(i(i(i(e,e),i(f,e6)),e7),e8))))),i(c,i(i(e6,e7),i(f,e8)))))))) | $ANS(TARG6).
-P(i(a,i(i(b,i(a,i(i(c,c),i(i(i(d,e),i(f,i(e6,i(i(e7,i(e6,i(i(e8,e8),i(f,d)))),i(e7,e))))),
  i(i(i(i(e9,e9),i(e10,e11)),e12),e13))))),i(b,i(i(e11,e12),i(e10,e13)))))) | $ANS(TARG6).
-P(i(i(a,i(i(i(b,c),i(d,i(i(e,i(d,b)),i(e,c)))),i(i(f,f),i(i(i(e6,e7),i(e8,i(e9,i(i(e10,i(e9,i(i(e11,e11),
  i(e8,e6)))),i(e10,e7))))),i(i(i(i(e12,e12),i(e13,e14)),e15),e16))))),i(a,i(i(e14,e15),
  i(e13,e16))))) | $ANS(TARG6).
-P(i(i(i(i(i(a,a),i(i(b,b),i(c,d))),e),f),i(i(d,e),i(c,f)))) | $ANS(TARG6).
-P(i(i(a,b),i(c,i(i(d,i(i(e,e),i(c,a))),i(d,b))))) | $ANS(TARG6).
-P(i(i(a,b),i(c,i(d,i(i(e,i(d,i(i(f,f),i(i(e6,e6),i(c,a))))),i(e,b)))))) | $ANS(TARG6).
-P(i(i(a,b),i(i(c,a),i(c,b)))) | $ANS(TARG6).
-P(i(a,i(i(b,i(i(c,c),i(a,i(d,e)))),i(b,i(i(f,d),i(f,e)))))) | $ANS(TARG6).
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-P(i(i(i(a,i(i(b,i(a,c)),i(b,c))),i(i(i(d,e),i(f,i(e6,i(i(e7,i(e6,i(i(e8,e8),i(f,d)))),i(e7,e))))),
  i(i(i(i(e9,e9),i(e10,e11)),e12),e13))),i(i(e11,e12),i(e10,e13)))) | $ANS(TARG6).
-P(i(i(a,i(i(i(b,c),i(i(d,b),i(d,c))),i(i(i(i(e,e),i(f,e6)),e7),e8))),i(a,i(i(e6,e7),i(f,e8))))) | $ANS(TARG6).
-P(i(i(a,i(i(i(i(b,b),i(i(c,c),i(d,e))),f),e6)),i(a,i(i(e,f),i(d,e6))))) | $ANS(TARG6).
-P(i(i(a,i(i(i(i(b,b),i(c,d)),e),f)),i(a,i(i(d,e),i(c,f))))) | $ANS(TARG6).
-P(i(i(a,b),i(i(c,i(i(d,d),i(i(e,e),f))),i(c,i(i(f,a),b))))) | $ANS(TARG6).
-P(i(i(i(a,a),i(b,c)),i(i(c,i(d,e)),i(b,i(i(f,d),i(f,e)))))) | $ANS(TARG6).
-P(i(i(a,b),i(i(c,d),i(c,i(i(d,a),b))))) | $ANS(TARG6).
-P(i(i(i(a,i(i(b,c),c)),i(d,e)),i(i(a,b),i(i(f,d),i(f,e))))) | $ANS(TARG6).
-P(i(i(a,i(i(b,b),i(c,d))),i(i(e,a),i(e,i(i(d,f),i(c,f)))))) | $ANS(TARG6).
-P(i(i(a,i(i(i(i(b,b),i(c,d)),d),e)),i(a,i(i(e,f),i(c,f))))) | $ANS(TARG6).
-P(i(i(a,b),i(i(i(i(i(c,c),i(d,a)),b),e),i(d,e)))) | $ANS(TARG6).
-P(i(i(a,i(b,c)),i(i(i(a,c),d),i(b,d)))) | $ANS(TARG6).
-P(i(i(a,i(b,c)),i(i(c,d),i(b,i(a,d))))) | $ANS(TARG6).
-P(i(i(a,i(i(b,b),i(c,d))),i(i(d,e),i(c,i(a,e))))) | $ANS(TARG6).
-P(i(i(i(i(a,b),i(c,b)),d),i(i(c,a),d))) | $ANS(TARG6).
-P(i(i(i(a,b),c),i(d,i(i(a,i(d,b)),c)))) | $ANS(TARG6).
-P(i(i(a,b),i(c,i(i(c,a),b)))) | $ANS(TARG6).
-P(i(i(a,b),i(i(i(i(b,c),c),d),i(a,d)))) | $ANS(TARG6).
-P(i(i(i(i(a,b),b),c),i(d,i(i(d,a),c)))) | $ANS(TARG6).
-P(i(i(a,b),i(c,i(i(c,i(b,d)),i(a,d))))) | $ANS(TARG6).
-P(i(i(i(i(a,b),i(i(c,a),b)),d),i(c,d))) | $ANS(TARG6).
-P(i(i(a,i(b,c)),i(d,i(i(d,b),i(a,c))))) | $ANS(TARG6).
-P(i(a,i(i(a,b),i(c,i(i(b,i(c,d)),d))))) | $ANS(TARG6).
-P(i(i(i(i(a,i(b,c)),c),d),i(b,i(a,d)))) | $ANS(TARG6).
-P(i(i(a,b),i(i(b,c),i(a,c)))) | $ANS(TARG6).
-P(i(i(i(a,i(i(a,b),c)),d),i(i(b,c),d))) | $ANS(TARG6).
-P(i(i(a,b),i(a,i(i(b,c),c)))) | $ANS(TARG6).
-P(i(i(a,i(b,c)),i(i(i(d,i(i(d,b),i(a,c))),e),e))) | $ANS(TARG6).
-P(i(i(a,i(i(b,b),c)),i(i(c,d),i(a,d)))) | $ANS(TARG6).
-P(i(i(i(i(i(a,a),i(b,c)),i(d,c)),e),i(i(d,b),e))) | $ANS(TARG6).
-P(i(a,i(i(i(b,b),i(c,d)),i(i(a,c),d)))) | $ANS(TARG6).
-P(i(i(a,i(b,c)),i(b,i(a,c)))) | $ANS(TARG6).
-P(i(i(i(a,a),i(b,c)),i(i(c,i(d,e)),i(b,i(d,e))))) | $ANS(TARG6).
-P(i(a,i(i(b,i(a,c)),i(b,c)))) | $ANS(TARG6).
-P(i(i(a,i(b,c)),i(i(i(d,d),i(e,a)),i(e,i(b,c))))) | $ANS(TARG6).
-P(i(i(i(a,a),i(b,i(c,d))),i(b,i(i(e,c),i(e,d))))) | $ANS(TARG6).
-P(i(i(a,i(i(b,b),c)),i(i(d,a),i(d,c)))) | $ANS(TARG6).
-P(i(i(a,i(b,c)),i(a,i(b,c)))) | $ANS(TARG6).
-P(i(i(a,i(i(b,b),i(c,d))),i(a,i(c,d)))) | $ANS(TARG6).
-P(i(i(i(a,a),i(b,c)),i(b,c))) | $ANS(TARG6).
-P(i(a,a)) | $ANS(TARG6).
%  Following negs of 72 singles, all of the known, for BCI.
-P(i(i(p,i(q,r)),i(i(i(s,s),i(t,q)),i(t,i(p,r))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(i(s,i(r,t)),i(q,i(s,t))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(r,r),i(i(i(q,s),s),t)),i(p,t)))) | $ANS(TARG2).
-P(i(i(p,q),i(i(q,i(i(r,r),i(s,t))),i(s,i(p,t))))) | $ANS(TARG2).
-P(i(i(i(i(i(p,p),i(q,r)),r),s),i(i(s,t),i(q,t)))) | $ANS(TARG2).
-P(i(i(i(i(p,q),q),r),i(i(i(s,s),i(r,t)),i(p,t)))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(i(i(i(s,s),p),r),t),i(q,t)))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(i(i(s,s),i(r,t)),i(q,i(p,t))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(i(i(q,r),r),s)),i(i(s,t),i(q,t)))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(s,i(i(r,i(s,t)),i(q,t))))) | $ANS(TARG2).
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-P(i(i(i(p,p),i(q,r)),i(i(i(i(r,s),s),t),i(q,t)))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(i(r,i(s,t)),i(s,i(q,t))))) | $ANS(TARG2).
-P(i(i(p,i(i(q,q),i(r,s))),i(i(s,t),i(r,i(p,t))))) | $ANS(TARG2).
-P(i(i(p,i(i(q,q),r)),i(s,i(i(r,i(s,t)),i(p,t))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(r,r),i(q,i(s,t))),i(s,i(p,t))))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(q,i(i(i(s,s),i(r,t)),i(p,t))))) | $ANS(TARG2).
-P(i(p,i(i(i(q,q),i(r,s)),i(i(s,i(p,t)),i(r,t))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(p,r)),i(i(i(s,s),i(r,t)),i(q,t))))) | $ANS(TARG2).
-P(i(i(p,q),i(r,i(i(i(s,s),i(q,i(r,t))),i(p,t))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(i(i(i(s,t),t),q),i(s,r)))) | $ANS(TARG2).
-P(i(p,i(i(i(q,q),i(r,i(p,s))),i(i(t,r),i(t,s))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,i(r,s))),i(i(t,q),i(r,i(t,s))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(r,r),i(i(i(s,p),q),t)),i(s,t)))) | $ANS(TARG2).
-P(i(i(p,i(i(q,q),i(r,s))),i(i(t,p),i(r,i(t,s))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(i(i(i(s,q),r),t),i(s,t)))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(i(s,i(t,q)),i(t,i(s,r))))) | $ANS(TARG2).
-P(i(i(p,i(i(q,q),r)),i(i(r,i(s,t)),i(s,i(p,t))))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(i(i(s,s),i(t,p)),i(q,i(t,r))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(i(i(q,r),i(i(s,q),r)),t)),i(s,t)))) | $ANS(TARG2).
-P(i(i(i(p,p),i(i(i(q,r),r),s)),i(i(t,q),i(t,s)))) | $ANS(TARG2).
-P(i(i(i(p,p),i(i(i(q,r),s),t)),i(i(r,s),i(q,t)))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,i(r,s))),i(i(s,t),i(r,i(q,t))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,i(r,s))),i(i(t,r),i(t,i(q,s))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,i(r,s))),i(r,i(i(s,t),i(q,t))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,i(r,s))),i(r,i(i(t,q),i(t,s))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,i(r,s))),i(t,i(i(t,r),i(q,s))))) | $ANS(TARG2).
-P(i(i(i(p,p),i(q,r)),i(q,i(i(s,i(r,t)),i(s,t))))) | $ANS(TARG2).
-P(i(i(i(p,p),q),i(i(i(i(q,i(r,s)),s),t),i(r,t)))) | $ANS(TARG2).
-P(i(i(p,i(i(q,q),i(r,s))),i(i(t,r),i(t,i(p,s))))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(i(s,i(i(t,t),p)),i(q,i(s,r))))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(q,i(i(i(s,s),i(t,p)),i(t,r))))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(s,i(i(i(t,t),i(s,q)),i(p,r))))) | $ANS(TARG2).
-P(i(i(p,i(q,r)),i(q,i(i(i(s,i(i(t,t),p)),i(s,r))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(i(i(r,r),i(s,t)),t),p),i(s,q)))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(i(q,r),r),i(i(s,s),t)),i(p,t)))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(i(r,p),q),i(i(s,s),t)),i(r,t)))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(r,r),i(s,i(q,t))),i(p,i(s,t))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(i(r,r),i(s,i(t,p))),i(t,i(s,q))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(r,i(i(s,s),i(q,t))),i(p,i(r,t))))) | $ANS(TARG2).
-P(i(i(p,q),i(i(r,i(i(s,s),i(t,p))),i(t,i(r,q))))) | $ANS(TARG2).
-P(i(i(p,q),i(p,i(i(i(r,r),i(s,i(q,t))),i(s,t))))) | $ANS(TARG2).
-P(i(i(p,q),i(r,i(i(i(s,s),i(t,i(r,p))),i(t,q))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(i(r,r),i(p,s))),i(i(s,t),i(q,t))))) | $ANS(TARG2).
-P(i(p,i(i(i(q,q),i(r,i(p,s))),i(i(s,t),i(r,t))))) | $ANS(TARG2).
-P(i(p,i(i(i(q,q),i(r,i(s,t))),i(i(p,s),i(r,t))))) | $ANS(TARG2).
-P(i(p,i(i(i(q,q),i(r,s)),i(i(t,i(p,r)),i(t,s))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(i(r,r),i(p,s))),i(i(t,q),i(t,s))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(i(r,r),i(s,t))),i(i(p,s),i(q,t))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(i(r,r),s)),i(i(s,i(p,t)),i(q,t))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(p,r)),i(i(i(s,s),i(t,q)),i(t,r))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(r,s)),i(i(i(t,t),i(p,r)),i(q,s))))) | $ANS(TARG2).
-P(i(p,i(i(q,i(p,r)),i(i(s,i(i(t,t),q)),i(s,r))))) | $ANS(TARG2).
-P(i(p,i(i(p,q),i(i(i(r,r),i(s,i(q,t))),i(s,t))))) | $ANS(TARG2).

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-P(i(p,i(i(q,r),i(i(i(s,s),i(r,i(p,t))),i(q,t))))) | $ANS(TARG2).
-P(i(p,i(i(p,q),i(i(r,i(i(s,s),i(q,t))),i(r,t))))) | $ANS(TARG2).
-P(i(p,i(i(q,r),i(i(r,i(i(s,s),i(p,t))),i(q,t))))) | $ANS(TARG2).
-P(i(p,i(i(q,r),i(i(s,i(i(t,t),i(p,q))),i(s,r))))) | $ANS(TARG2).
-P(i(i(i(i(i(p,p),i(q,r)),r),s),i(i(t,q),i(t,s)))) | $ANS(TARG2).
-P(i(i(i(i(i(p,p),q),r),s),i(i(q,i(t,r)),i(t,s)))) | $ANS(TARG2).
-P(i(i(i(i(p,i(q,r)),r),s),i(i(i(t,t),p),i(q,s)))) | $ANS(TARG2).
-P(i(i(i(i(p,q),q),r),i(i(i(s,s),i(t,p)),i(t,r)))) | $ANS(TARG2).
-P(i(i(i(i(p,q),r),s),i(i(i(t,t),i(q,r)),i(p,s)))) | $ANS(TARG2).
-P(i(a,a)) | $ANS(I).
-P(i(i(a1,i(b,a2)),i(b,i(a1,a2)))) | $ANS(C).  %  C
-P(i(i(a1,b),i(i(a2,a1),i(a2,b)))) | $ANS(B).  % B
end_of_list.

list(hints).
%  Following 52 prove from cand48 join of BCI, .out1b8.
P(i(i(x,y),i(z,i(i(u,i(z,x)),i(u,y))))).
P(i(i(x,y),i(z,i(u,i(i(v,i(u,i(i(w,w),i(z,x)))),i(v,y)))))).
P(i(x,i(i(y,i(x,i(i(i(i(z,z),i(u,v)),w),v6))),i(y,i(i(v,w),i(u,v6)))))).
P(i(x,i(y,i(i(z,i(y,i(i(u,u),i(x,i(i(i(i(v,v),i(w,v6)),v7),v8))))),i(z,i(i(v6,v7),i(w,v8))))))).
P(i(x,i(i(y,i(x,i(i(z,z),i(i(i(u,v),i(w,i(v6,i(i(v7,i(v6,i(i(v8,v8),i(w,u)))),i(v7,v))))),
  i(i(i(i(v9,v9),i(v10,v11)),v12),v13))))),i(y,i(i(v11,v12),i(v10,v13)))))).
P(i(i(x,i(i(i(y,z),i(u,i(i(v,i(u,y)),i(v,z)))),i(i(w,w),i(i(i(v6,v7),i(v8,i(v9,i(i(v10,i(v9,i(i(v11,v11),
  i(v8,v6)))),i(v10,v7))))),i(i(i(i(v12,v12),i(v13,v14)),v15),v16))))),i(x,i(i(v14,v15),i(v13,v16))))).
P(i(i(i(i(i(x,x),i(i(y,y),i(z,u))),v),w),i(i(u,v),i(z,w)))).
P(i(i(x,y),i(z,i(i(u,i(i(v,v),i(z,x))),i(u,y))))).
P(i(i(x,y),i(z,i(u,i(i(v,i(u,i(i(w,w),i(i(v6,v6),i(z,x))))),i(v,y)))))).
P(i(i(x,y),i(i(z,x),i(z,y)))).
P(i(x,i(i(y,i(i(z,z),i(x,i(u,v)))),i(y,i(i(w,u),i(w,v)))))).
P(i(i(i(x,i(i(y,i(x,z)),i(y,z))),i(i(i(u,v),i(w,i(v6,i(i(v7,i(v6,i(i(v8,v8),i(w,u)))),
  i(v7,v))))),i(i(i(i(v9,v9),i(v10,v11)),v12),v13))),i(i(v11,v12),i(v10,v13)))).
P(i(i(x,i(i(i(y,z),i(i(u,y),i(u,z))),i(i(i(i(v,v),i(w,v6)),v7),v8))),i(x,i(i(v6,v7),i(w,v8))))).
P(i(i(x,i(i(i(i(y,y),i(i(z,z),i(u,v))),w),v6)),i(x,i(i(v,w),i(u,v6))))).
P(i(i(x,i(i(i(i(y,y),i(z,u)),v),w)),i(x,i(i(u,v),i(z,w))))).
P(i(i(x,y),i(i(z,i(i(u,u),i(i(v,v),w))),i(z,i(i(w,x),y))))).
P(i(i(i(x,x),i(y,z)),i(i(z,i(u,v)),i(y,i(i(w,u),i(w,v)))))).
P(i(i(x,y),i(i(z,u),i(z,i(i(u,x),y))))).
P(i(i(i(x,i(i(y,z),z)),i(u,v)),i(i(x,y),i(i(w,u),i(w,v))))).
P(i(i(x,i(i(y,y),i(z,u))),i(i(v,x),i(v,i(i(u,w),i(z,w)))))).
P(i(i(x,i(i(i(i(y,y),i(z,u)),u),v)),i(x,i(i(v,w),i(z,w))))).
P(i(i(x,y),i(i(i(i(i(z,z),i(u,x)),y),v),i(u,v)))).
P(i(i(x,i(y,z)),i(i(i(x,z),u),i(y,u)))).
P(i(i(x,i(y,z)),i(i(z,u),i(y,i(x,u))))).
P(i(i(x,i(i(y,y),i(z,u))),i(i(u,v),i(z,i(x,v))))).
P(i(i(i(i(x,y),i(z,y)),u),i(i(z,x),u))).
P(i(i(i(x,y),z),i(u,i(i(x,i(u,y)),z)))).
P(i(i(x,y),i(z,i(i(z,x),y)))).
P(i(i(x,y),i(i(i(i(y,z),z),u),i(x,u)))).
P(i(i(i(i(x,y),y),z),i(u,i(i(u,x),z)))).
P(i(i(x,y),i(z,i(i(z,i(y,u)),i(x,u))))).
P(i(i(i(i(x,y),i(i(z,x),y)),u),i(z,u))).
P(i(i(x,i(y,z)),i(u,i(i(u,y),i(x,z))))).
P(i(x,i(i(x,y),i(z,i(i(y,i(z,u)),u))))).
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P(i(i(i(i(x,i(y,z)),z),u),i(y,i(x,u)))).
P(i(i(x,y),i(i(y,z),i(x,z)))).
P(i(i(i(x,i(i(x,y),z)),u),i(i(y,z),u))).
P(i(i(x,y),i(x,i(i(y,z),z)))).
P(i(i(x,i(y,z)),i(i(i(u,i(i(u,y),i(x,z))),v),v))).
P(i(i(x,i(i(y,y),z)),i(i(z,u),i(x,u)))).
P(i(i(i(i(i(x,x),i(y,z)),i(u,z)),v),i(i(u,y),v))).
P(i(x,i(i(i(y,y),i(z,u)),i(i(x,z),u)))).
P(i(i(x,i(y,z)),i(y,i(x,z)))).
P(i(i(i(i(x,x),i(y,z)),i(i(z,i(u,v)),i(y,i(u,v))))).
P(i(x,i(i(y,i(x,z)),i(y,z)))).
P(i(i(x,i(y,z)),i(i(i(u,u),i(v,x)),i(v,i(y,z))))).
P(i(i(i(x,x),i(y,i(z,u))),i(y,i(i(v,z),i(v,u))))).
P(i(i(x,i(i(y,y),z)),i(i(u,x),i(u,z)))).
P(i(i(x,i(y,z)),i(x,i(y,z)))).
P(i(i(x,i(i(y,y),i(z,u))),i(x,i(z,u)))).
P(i(i(i(i(x,x),i(y,z)),i(y,z))).
P(i(x,x)).
%  Following 23/16 prove a known single from cand48, temp.halleck.bci.cand48.out1b5,
%  of int in that vars17 length68?.
P(i(i(x,y),i(z,i(i(u,i(z,x)),i(u,y))))).
P(i(i(x,y),i(z,i(u,i(i(v,i(u,i(i(w,w),i(z,x)))),i(v,y)))))).
P(i(x,i(i(y,i(x,i(i(i(i(z,z),i(u,v)),w),v6))),i(y,i(i(v,w),i(u,v6)))))).
P(i(x,i(y,i(i(z,i(y,i(i(u,u),i(x,i(i(i(i(v,v),i(w,v6)),v7),v8))))),i(z,i(i(v6,v7),i(w,v8)))))).
P(i(x,i(i(y,i(x,i(i(z,z),i(i(i(u,v),i(w,i(v6,i(i(v7,i(v6,i(i(v8,v8),i(w,u)))),i(v7,v))))),
  i(i(i(i(v9,v9),i(v10,v11)),v12),v13))))),i(y,i(i(v11,v12),i(v10,v13)))))).
P(i(i(x,i(i(i(y,z),i(u,i(i(v,i(u,y)),i(v,z)))),i(i(w,w),i(i(i(v6,v7),i(v8,i(v9,i(i(v10,i(v9,i(i(v11,v11),
  i(v8,v6)))),i(v10,v7))))),i(i(i(i(v12,v12),i(v13,v14)),v15),v16))))),i(x,i(i(v14,v15),i(v13,v16))))).
P(i(i(i(i(i(x,x),i(i(y,y),i(z,u))),v),w),i(i(u,v),i(z,w)))).
P(i(i(x,y),i(z,i(i(u,i(i(v,v),i(z,x))),i(u,y))))).
P(i(i(x,y),i(z,i(u,i(i(v,i(u,i(i(w,w),i(i(v6,v6),i(z,x))))),i(v,y)))))).
P(i(i(x,y),i(i(z,x),i(z,y)))).
P(i(i(i(x,i(i(y,i(x,z)),i(y,z))),i(i(i(u,v),i(w,i(v6,i(i(v7,i(v6,i(i(v8,v8),i(w,u)))),
  i(v7,v))))),i(i(i(i(v9,v9),i(v10,v11)),v12),v13))),i(i(v11,v12),i(v10,v13)))).
P(i(i(x,i(i(i(y,z),i(i(u,y),i(u,z))),i(i(i(i(v,v),i(w,v6)),v7),v8))),i(x,i(i(v6,v7),i(w,v8))))).
P(i(i(x,i(i(i(i(y,y),i(i(z,z),i(u,v))),w),v6)),i(x,i(i(v,w),i(u,v6))))).
P(i(i(x,i(i(i(i(y,y),i(z,u)),v),w)),i(x,i(i(u,v),i(z,w))))).
P(i(x,i(i(y,i(i(z,z),i(x,i(u,v))))),i(y,i(i(w,u),i(w,v))))).
P(i(i(x,y),i(i(z,i(i(u,u),i(i(v,v),w))),i(z,i(i(w,x),y))))).
P(i(i(i(x,x),i(y,z)),i(i(z,i(u,v)),i(y,i(i(w,u),i(w,v))))).
P(i(i(x,y),i(i(z,u),i(z,i(i(u,x),y))))).
P(i(i(i(x,i(i(y,z),z)),i(u,v)),i(i(x,y),i(i(w,u),i(w,v))))).
P(i(i(x,i(i(y,y),i(z,u))),i(i(v,x),i(v,i(i(u,w),i(z,w)))))).
P(i(i(x,i(i(i(i(y,y),i(z,u)),u),v)),i(x,i(i(v,w),i(z,w))))).
P(i(i(x,y),i(i(i(i(i(z,z),i(u,x)),y),v),i(u,v)))).
P(i(i(x,i(i(y,y),i(z,u))),i(i(u,v),i(z,i(x,v))))).
end_of_list.

list(demodulators).
(P(i(i(i(i(i(x,x),i(y,z)),i(y,u)),v),i(i(z,u),v))) = junk).
%  %  (i(i(x,x),y) = junk).
%  %  (i(y,i(x,x)) = junk).
(i(x,junk) = junk).
```

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(i(junk,x) = junk).
(P(junk) = $T).
end_of_list.
```

Here you have an example of concentrating on finding a proof with less variable richness and succeeding and, also, finding as a side effect a proof with far less formula complexity. On the other hand, increasing the value assigned to max_weight or to max_distinct_vars or such can lead to finding a proof more pleasing or, occasionally, finding a first proof that would otherwise have been missed. Yes, all is so complicated.

The given proof brought utter joy to me. After all, an examination of the proof reveals a sharp reduction in formula complexity, 64 compared with the 220 found in the first proof showing candidate 48 to be a (new) single axiom. What a charming story, one (whose first chapter) began with Ulrich supplying (on his website) numerous candidates for being a new shortest single axiom for *BCI*. The second chapter that commenced with a request from Halleck to find a shorter proof, the details of which are found in a notebook to be placed on my website (automatedreasoning.net), a notebook entitled bci.revisited. In the bci.revisited notebook, you read about the Halleck discovery of a proof establishing candidate formula 29 to be a new single axiom, and you learn of Ulrich's success with adding forty-three new single axioms. Here, as the story unfolds and the chapters are read, you learn of Halleck's dispatching in the affirmative of candidate 18 and of candidate 10, of my dispatching of candidates 23, 51, and 46, and of Stickel's dispatching in the affirmative of candidates 42, 48, and 19. Only a bit remains, none of which appears to be promising.

```
% Following 5 candidates for being a single axiom for BCI, not yet classified, 05-17-09.
P(i(i(u,i(i(v,v),w)),i(i(i(i(x,y),y),u),i(x,w)))).  % BCI-Candidate 13, unlikely i(x,x) variants
P(i(i(i(i(u,v),v),i(i(w,w),x)),i(i(x,y),i(u,y)))).  % BCI-Candidate 52
P(i(i(i(i(u,v),v),i(i(w,w),x)),i(i(y,u),i(y,x)))).  % BCI-Candidate 53, unlikely i(x,x) variants
P(i(i(i(i(u,v),w),i(i(x,x),y)),i(i(v,w),i(u,y)))).  % BCI-Candidate 54, unlikely i(x,x) variants
P(i(i(i(i(u,v),v),w),i(i(w,i(i(x,x),y)),i(u,y)))).  % BCI-Candidate 57, unlikely i(x,x) variants
```

Finally, of a global nature, Overbeek's request is met: You learn of the successful use of proof-shortening methodologies in the context of finding first proofs, settling conjectures, and answering open questions.