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Hunting Treasure in Intuitionism

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1. The Nature of the Hunt

As you may have read in earlier notebooks found on my website (automatedreasoning.net), treasure hunts take many forms, with various rewards, and (often) with diverse clues about how to proceed. Mathematicians and logicians, among others, often seek treasure in the form of proofs, sometimes a first proof of a purported theorem, sometimes a shorter proof than is found in the literature, and sometimes a proof that avoids the use of some thought-to-be indispensable lemma. A published paper or a book can offer a sequence of theorems, where various proofs rely on results found earlier in the work. Now, what would you do if asked to prove a collection of theorems—in this notebook, a collection consisting of sixty-three items—and, further, asked to find those sixty-three proofs with heavy use of an automated reasoning program? (The sixty-three include eight of Heyting's 11-axiom system; his fifth, seventh, and tenth are respectively axioms 1, 6, and 10 of the Horn system used here and, therefore, do not need proving.) In this notebook, you will visit the labyrinth that I traveled as I attempted to answer such a question.

I was asked to do so by my valued colleague Michael Beeson. The area of concern was intuitionism, and the axiom system Beeson chose consisted of ten axioms (to be given shortly) by Horn. An additional requirement on the search was placed by me, namely, to produce a systematic approach that might be of use to future treasure hunters who were after a (possibly) tightly connected sequence of proofs. (If the methodology was found, I thought it might be of use in preparing a paper for publication or a chapter of a book to be written, where no proofs were, at the time, available.) This notebook features the proofs and the methodology I finally discovered. I do not in any way offer what I would call an algorithm for finding a sequence of proofs. Further, I strongly suspect that no algorithm of the kind exists; after all, mathematics and logic are deep subjects.

As you accompany me on this journey through yet another land of research, I must tell you that the road is far from straight, but you may enjoy trying to anticipate what is to come next. The journey lasted more than five months, with sometimes long periods of inattention, and consisted of a large number of experiments; its length can be explained in part by my greed for finding what might be termed artificial goals for proof length. (I also, in keeping with science, note that this notebook may not record the exact details of the trip; but I shall try.) The last part of the trip—which occurred after the desired sixty-three proofs were found, as well as a systematic approach for completing each—was concerned with (as is so typical in my research) the discovery of "short" proofs, some of which were very difficult to find under the conditions I imposed.

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Before displaying the sixty-three target theorems, I pause to address an implied question that recently (in mid-2010) was the focus of a rather large number of e-mail messages. This question may also have already occurred to you. Why should effort and time be expended in search of proofs that already exist, or proofs that can be easily proved by hand, where the proofs are obtained with an automated reasoning program? The wellspring for this question was a challenge I offered in an AAR newsletter, namely, to prove with an automated reasoning program from a set of four simple axioms a 93-symbol formula (due to Rezus). Since this proof could easily be met by hand, without recourse to automation, some of the newsletter readers wanted to know why I had bothered to issue such a challenge. The key element was, in fact, the use of a reasoning program. The main obstacle rested with the fact that, typically, reasoning programs (as well as unaided researchers) gravitate toward simple targets, certainly not a target formula of length 93. Yet by meeting challenges of that type—and the type in focus here—the researcher in automated reasoning often develops a new methodology or a new strategy or extends what already exists, with the result that the power of reasoning programs is sharply increased. With that increased power, theorems that had been out of reach are brought within reach. The consequences can be startling and delightful. Among them are the answering of questions that had been open for decades, the discovery that a thought-to-be axiom is in fact unneeded, the finding of a proof that avoids the use of a lemma conjectured to be indispensable, and the completion of a proof far more elegant than previously known. From history, you may recall that the successful seeking of a proof of the classroom exercise asserting that commutativity for groups can be proved in the presence of the added equation $xx = x$ led directly to the formation of the set of support strategy. Also, the subformula strategy (if memory serves) was formulated because of an attempt to prove with William McCune's program OTTER the Rezus 93-symbol formula. Further evidence of the value of seeking proofs with a program such as OTTER, where years earlier proofs were found of theorems easily proved by hand, is provided by the Ulrich-Wos success in equivalential calculus with settling the status, in the affirmative, of the formula *XCB* in the context of being a single axiom for that area of logic. The conclusion that can correctly be drawn is that access to proofs produced by hand or proofs already in the literature in no way sheds light on how such proofs can be found by a reasoning program. Proofs of the theorems studied here were already available in the literature.

And so, let us now embark on the journey motivated by Beeson.

When undertaking a long journey that presents serious obstacles to overcome, one must have provisions in order to survive. For the journey featured here, my provisions included many years of experimentation with OTTER. They also included—so important to my research—excitement at seeking to find a methodology that would be at least in part new to me. In this case, there existed the prospect of finding some proofs shorter than discovered early in the hunt. My final provision: Beeson supplied the following ten (Horn) axioms, where the functions *i* denotes implication, *n* denotes negation, *a* denotes logical **or**, *k* denotes logical **and**, and the predicate *P* denotes "is provable". The means of transportation through the land of intuitionism is the inference rule condensed detachment, the following clause in which "-" denotes logical **not** and " | " denotes logical **or**.

```
-P(i(x,y)) | -P(x) | P(y).

% Following 10 axiomatize intuitionistic logic from Horn's 1962 paper
P(i(x,i(y,x))).     % 1
P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).   % 2
P(i(k(x,y),x)).     % 3
P(i(k(x,y),y)).     % 4
P(i(i(x,y),i(i(x,z),i(x,k(y,z))))).   % 5
P(i(x,a(x,y))).          % 6
P(i(x,a(y,x))).          % 7
P(i(i(x,y),i(i(z,y),i(a(x,z),y)))). % 8
P(i(i(x,n(y)),i(y,n(x)))).   % 9
P(i(n(x),i(x,y))).      % 10
```

The approach featured in this notebook is iterative, one experiment building on results obtained in earlier experiments. The theorems are taken from a 1962 paper by Horn. the following sixty-three targets are presented in negated form, some arising from if-and-only-if theorems to be proved. (When a percent sign appears, from that point on to the end of the line, OTTER treats what occurs as a comment.)

```
% Following are eight of Heyting's axioms and other theorems to prove.
%The 5th, 7th, and 10th are omitted: they are axioms 1, 6, and 10 of the Horn system.
-P(i(b,k(b,b))) | $ANS(01).
-P(i(k(b,c),k(c,b))) | $ANS(02).
-P(i(i(b,c),i(k(b,d),k(c,d)))) | $ANS(03).
-P(i(k(i(b,c),i(c,d)),i(b,d))) | $ANS(04).
-P(i(k(b,i(b,c)),c)) | $ANS(06).
-P(i(a(b,c),a(c,b))) | $ANS(08).
-P(i(k(i(b,d),i(c,d)),i(a(b,c),d))) | $ANS(09).
-P(i(k(i(b,c),i(b,n(c))),n(b))) | $ANS(11).
-P(i(i(q,r),i(i(p,q),i(p,r)))) | $ANS(H2).
-P(i(i(p,i(q,r)),i(q,i(p,r)))) | $ANS(H3).
-P(i(i(p,n(p)),n(p)))          | $ANS(N1).
-P(i(p,i(n(p),q))) | $ANS(N2).
% Following theorems from Heyting's paper.
-P(i(p,p)) | $ANS(H221).
-P(i(k(i(p,q),i(r,d)),i(k(p,r),k(q,d)))) | $ANS(H223).
% -P(iff(k(i(p,q),i(p,r)),i(p,k(q,r)))) | $ANS(H224).
-P(i(k(p,i(q,r)),i(q,k(p,r)))) | $ANS(H225).
-P(i(p,i(q,k(q,p)))) | $ANS(H226).
% -P(iff(i(p,i(b,c)),i(k(p,b),c))) | $ANS(H227).
% -P(iff(i(p,i(b,c)),i(b,i(p,c)))) | $ANS(H2271).
-P(i(i(p,c),i(k(p,b),c))) | $ANS(H228).
-P(i(i(p,b),i(p,i(c,b)))) | $ANS(H2281).
-P(i(k(p,i(k(p,b),c)),i(b,c))) | $ANS(H2282).
-P(i(i(p,b),i(i(b,c),i(p,c)))) | $ANS(H229).
-P(i(i(b,c),i(i(p,b),i(p,c)))) | $ANS(H2291).
-P(i(k(k(p,b),c),k(p,k(b,c)))) | $ANS(H23).
-P(i(k(k(p,b),c),k(k(b,p),c))) | $ANS(H231).
-P(i(k(p,k(b,c)),k(k(p,b),c))) | $ANS(H232).
-P(i(k(k(i(p,b),k(b,c)),i(c,d)),i(p,d))) | $ANS(H24).
-P(i(a(a(p,b),c),a(p,a(b,c)))) | $ANS(H32).
-P(i(a(p,a(b,c)),a(a(p,b),c))) | $ANS(H321).
-P(i(a(p,p),p))            | $ANS(H322).
-P(i(k(i(p,b),i(c,d)),i(a(p,c),a(b,d)))) | $ANS(H33).
-P(i(i(p,b),i(k(p,c),a(b,d)))) | $ANS(H331).
-P(i(i(p,b),i(a(p,b),b)))    | $ANS(H332).
-P(i(i(a(p,b),b),i(p,b)))    | $ANS(H333).
-P(i(i(p,b),i(a(p,c),a(b,c)))) | $ANS(H334).
-P(i(a(a(p,b),c),a(a(b,p),c))) | $ANS(H335).
-P(i(a(a(b,p),c),a(a(p,c),b))) | $ANS(H3351).
-P(i(a(k(p,c),k(b,c)),k(a(p,b),c))) | $ANS(H34).
-P(i(k(a(p,b),c),a(k(p,c),k(b,c)))) | $ANS(H341).
-P(i(a(k(p,b),c),k(a(p,c),a(b,c)))) | $ANS(H3421).
-P(i(k(a(p,c),a(b,c)),a(k(p,b),c))) | $ANS(H3422).
% -P(iff(a(k(p,b),c),k(a(p,c),a(b,c)))) | $ANS(H342).
-P(i(k(k(i(p,a(b,c)),i(b,d)),i(c,e)),i(p,a(d,e)))) | $ANS(H35).
-P(i(k(k(i(p,a(b,c)),i(b,d)),i(c,d)),i(p,d))) | $ANS(H351).
-P(i(a(p,b),i(i(p,b),b)))  | $ANS(H36).
-P(i(i(p,b),i(n(b),n(p)))) | $ANS(H42).
-P(i(i(p,b),i(n(n(p)),n(n(b))))) | $ANS(H422).
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-P(i(i(k(p,b),c),i(k(p,n(c)),n(b)))) | $ANS(H423).
-P(i(k(i(p,b),n(b)),n(p))) | $ANS(H424).
-P(i(p,n(n(p)))) | $ANS(H43).
-P(i(n(p),n(n(n(p))))) | $ANS(H431).
-P(i(n(n(n(p))),n(p))) | $ANS(H432).
-P(i(k(p,n(p)),b)) | $ANS(H44).
-P(i(a(k(p,n(p)),b),b)) | $ANS(H441).
-P(i(k(a(p,b),n(p)),b)) | $ANS(H442).
-P(i(n(p),i(n(b),n(a(p,b))))) | $ANS(H443).
-P(i(n(a(p,b)),k(n(p),n(b)))) | $ANS(H4441).
-P(i(k(n(p),n(b)),n(a(p,b)))) | $ANS(H4442).
% -P(iff(n(a(p,b)),k(n(p),n(b)))) | $ANS(H444).
-P(i(a(p,n(p)),i(n(n(p)),p))) | $ANS(H445).
-P(i(a(n(p),b),i(p,b))) | $ANS(H446).
-P(i(a(p,b),i(n(p),b))) | $ANS(H447).
% Following 5 are expanded theorems corresponding to the iff theorems.
-P(k(i(k(i(p,q),i(p,r)),i(p,k(q,r))),i(i(p,k(q,r)),k(i(p,q),i(p,r))))) | $ANS(HIF1).
-P(k(i(i(p,i(q,r)),i(k(p,q),r)),i(i(k(p,q),r),i(p,i(q,r))))) | $ANS(HIF2).
-P(k(i(i(p,i(q,r)),i(q,i(p,r))),i(i(q,i(p,r)),i(p,i(q,r))))) | $ANS(HIF3).
-P(k(i(a(k(p,q),r),k(a(p,r),a(q,r))),i(k(a(p,r),a(q,r)),a(k(p,q),r)))) | $ANS(HIF4).
-P(k(i(n(a(p,q)),k(n(p),n(q))),i(k(n(p),n(q)),n(a(p,q))))) | $ANS(HIF5).
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2. Some History, Background, and a Beginning

The study by Beeson and me began in the summer of 2008. One year later, three theorems still remained unproved: H341, H3422, and HIF4. As it turned out, even quite a while after the experiments were being conducted in search of a methodology, these three theorems resisted proof. Beeson suggested we could find their proofs by studying each with a Gentzen-style approach, followed by mapping the proofs so obtained into the style desired, the type of proof OTTER finds. I, as you would predict, wished a direct approach that would produce a method for finding the sixty-three proofs with OTTER.

I therefore began a far more systematic attack that, I hoped, would produce a methodical treatment eventually yielding sixty-three proofs. What I report here are the experiments conducted in 2010, experiments that resemble (but not precisely) experiments conducted in 2009 and, perhaps, in 2008. The spirit of those early experiments is, however, indeed captured. Before supplying the input file for those experiments, however, I shall discuss a feew key aspects of OTTER.

In part because a huge number of experiments had failed to obtain all of the desired sixty-three proofs, I conjectured that the first item to address was strategy, both to restrict OTTER's reasoning and to direct it. I could restrict the program's reasoning by placing limits of various kinds, such as on the number of symbols present in a newly deduced and retained item and on the number of distinct variables allowed in such an item. I could also restrict the reasoning by placing some of the ten Horn axioms in the initial) list(usable); such items are not allowed to initiate applications of an inference rule. In fact, in the beginning, I placed axioms 1-4 (of the Horn set) in (the initial) list(sos), which allowed the program to use any of the four to initiate lines of reasoning. Axioms 9 and 10 were commented out with a percent sign; the notion was to avoid focusing on negation at the start. (I expected to remove this restriction after a while, being aware that I would iterate, having later runs rely on results of earlier runs, by including proof steps of completed proofs in one capacity or another.) I assigned the value 4 to max_distinct_vars, preventing retention of a new item that relied on five or more distinct variables. I assigned the value 12 to max_weight, which in general had the program discard new items that relied on thirteen symbols or more. However, I had the program change that value to 10 after 100 items were chosen to initiate inference-rule application. The decision to reduce the max_weight is usually promoted by the conjecture that, if not done so, the program will drown; this is especially true with a breadth-first, or level-saturation, search, which is the choice I made. For the last important restriction, I instructed OTTER to immediately discard a new deduction when it contained a term of the form $n(n(t))$ for some term t. Yes, I admit that these restrictions resulted from guesses; indeed, from what I know, no algorithm exists for choosing with certainty what is effective.

Years of experimentation made it clear to me that, in addition to restricting the reasoning, it must be also directed. I chose to rely in the beginning on the *subformula strategy*, formulated by me in late 2007 (if memory serves) in the study of a Rezus-style 93-symbol single axiom for an area known as BCI logic (see the notebook **An Astonishing Discovery from Proof Shortening: The Fecundity of the BCI Logic***). Briefly, for the subformula strategy, you select some set of formulas (or equations) from the problem and extract all of the nontrivial subformulas (or expressions) to be used to guide the program. To each such item, you assign a small value. Most of the selected subformulas are proper, but I usually include the entire formula as well. Similar to remarks (made in Section 4) concerning resonators, the subformulas are included with the intention that their use will sharply promote success by effectively directing the reasoning of the program in use. When seeking where next to focus its attention for conclusion drawing, the program will be directed to formulas (or equations) that are now assigned values smaller than they would otherwise have been assigned. In other words, preference is given to (entire) formulas or equations that contain a chosen (and found in the input file) subformula. (For the historian, and relevant to a remark made earlier in this notebook, the subformula strategy was formulated directly because of a successful attempt to prove, with OTTER, a Rezus 93-symbol formula, even though that formula was easily proved by hand.) From the ten Horn axioms, 40 nontrivial subformulas were extracted and used, and from the sixty-three target theorems, 219 nontrivial subformulas were chosen for inclusion. As you see, the idea was to have the program emphasize drawn conclusions that in part relied on one or more subformulas from the Horn axioms or from the target theorems.

I now offer you the first input file I used as I began my most lengthy journey in search of sixty-three proofs. The input file reflects the remarks just made about choices regarding strategy and the like. The journey—with its many twists and turns—proved even longer than need be because I added the goal of finding short proofs for each of the sixty-three. But the effort was worthwhile, for it led not onnly to the sought-after proofs but also to a systematic approach for proving a sequence of related theorems.

Input File 1 for the Study of Some Theorems from Intuitionism

```
% Otter file, by Beeson and Wos
% Intuitionistic propositional logic with axioms from Horn 1962
% goal to derive 63 theses from Heyting 1930.

set(hyper_res).
assign(max_weight,12).
assign(change_limit_after,100).
assign(new_max_weight,10).
clear(print_kept).
%  set(process_input).
%  set(ancestor_subsume).
set(back_sub).
clear(print_back_sub).
assign(max_distinct_vars,4).
%  assign(pick_given_ratio,2).
assign(max_mem,1900000).
%  assign(max_seconds,1).
%  assign(max_given,24).
assign(max_proofs,-1).
assign(report,5400).
set(order_history).
set(input_sos_first).
set(sos_queue).
```

```
weight_list(pick_and_purge).
weight(a(a($(1),$(1)),$(1)),1000).
weight(k(k($(1),$(1)),$(1)),1000).
weight(a(k(x,y),z),-5).
weight(k(a(x,y),a(u,v)),-4).
% Following 40 subformulas from the Horn formulas.
weight(P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))),2).
weight(P(i(i(x,n(y)),i(y,n(x)))),2).
weight(P(i(i(x,y),i(i(x,z),i(x,k(y,z))))),2).
weight(P(i(i(x,y),i(i(z,y),i(a(x,z),y)))),2).
weight(P(i(k(x,y),x)),2).
weight(P(i(k(x,y),y)),2).
weight(P(i(n(x),i(x,y))),2).
weight(P(i(x,a(x,y))),2).
weight(P(i(x,a(y,x))),2).
weight(P(i(x,i(y,x))),2).
weight(a(x,y),2).
weight(a(x,z),2).
weight(a(y,x),2).
weight(i(a(x,z),y),2).
weight(i(i(x,i(y,z)),i(i(x,y),i(x,z))),2).
weight(i(i(x,n(y)),i(y,n(x))),2).
weight(i(i(x,y),i(i(x,z),i(x,k(y,z)))),2).
weight(i(i(x,y),i(i(z,y),i(a(x,z),y))),2).
weight(i(i(x,y),i(x,z)),2).
weight(i(i(x,z),i(x,k(y,z))),2).
weight(i(i(z,y),i(a(x,z),y)),2).
weight(i(k(x,y),x),2).
weight(i(k(x,y),y),2).
weight(i(n(x),i(x,y)),2).
weight(i(x,a(x,y)),2).
weight(i(x,a(y,x)),2).
weight(i(x,i(y,x)),2).
weight(i(x,i(y,z)),2).
weight(i(x,k(y,z)),2).
weight(i(x,n(y)),2).
weight(i(x,y),2).
weight(i(x,z),2).
weight(i(y,n(x)),2).
weight(i(y,x),2).
weight(i(y,z),2).
weight(i(z,y),2).
weight(k(x,y),2).
weight(k(y,z),2).
weight(n(x),2).
weight(n(y),2).
% Following 219 are sorted subformulas of the 63 targets.
weight(P(i(a(a(x,x),y),a(a(x,x),y))),2).
weight(P(i(a(a(x,x),y),a(a(x,y),x))),2).
weight(P(i(a(a(x,x),y),a(x,a(x,y)))),2).
weight(P(i(a(k(x,n(x)),x),x)),2).
weight(P(i(a(k(x,x),y),k(a(x,y),a(x,y)))),2).
weight(P(i(a(k(x,y),k(x,y)),k(a(x,x),y))),2).
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weight(P(i(a(n(x),x),i(x,x))),2).
weight(P(i(a(x,a(x,y)),a(a(x,x),y))),2).
weight(P(i(a(x,n(x)),i(n(n(x)),x))),2).
weight(P(i(a(x,x),i(i(x,x),x))),2).
weight(P(i(a(x,x),i(n(x),x))),2).
weight(P(i(a(x,x),x)),2).
weight(P(i(a(x,y),a(y,x))),2).
weight(P(i(i(a(x,x),x),i(x,x))),2).
weight(P(i(i(k(x,x),y),i(k(x,n(y)),n(x)))),2).
weight(P(i(i(x,i(y,z)),i(y,i(x,z)))),2).
weight(P(i(i(x,n(x)),n(x))),2).
weight(P(i(i(x,x),i(a(x,x),x))),2).
weight(P(i(i(x,x),i(a(x,y),a(x,y)))),2).
weight(P(i(i(x,x),i(i(x,y),i(x,y)))),2).
weight(P(i(i(x,x),i(k(x,y),a(x,z)))),2).
weight(P(i(i(x,x),i(n(n(x)),n(n(x))))),2).
weight(P(i(i(x,x),i(n(x),n(x)))),2).
weight(P(i(i(x,x),i(x,i(y,x)))),2).
weight(P(i(i(x,y),i(i(x,x),i(x,y)))),2).
weight(P(i(i(x,y),i(k(x,x),y))),2).
weight(P(i(i(x,y),i(k(x,z),k(y,z)))),2).
weight(P(i(i(y,z),i(i(x,y),i(x,z)))),2).
weight(P(i(k(a(x,x),n(x)),x)),2).
weight(P(i(k(a(x,x),y),a(k(x,y),k(x,y)))),2).
weight(P(i(k(a(x,y),a(x,y)),a(k(x,x),y))),2).
weight(P(i(k(i(x,x),i(y,z)),i(a(x,y),a(x,z)))),2).
weight(P(i(k(i(x,x),n(x)),n(x))),2).
weight(P(i(k(i(x,y),i(x,n(y))),n(x))),2).
weight(P(i(k(i(x,y),i(y,z)),i(x,z))),2).
weight(P(i(k(i(x,y),i(z,z)),i(k(x,z),k(y,z)))),2).
weight(P(i(k(i(x,z),i(y,z)),i(a(x,y),z))),2).
weight(P(i(k(k(i(x,a(x,y)),i(x,z)),i(y,u)),i(x,a(z,u)))),2).
weight(P(i(k(k(i(x,a(x,y)),i(x,z)),i(y,z)),i(x,z))),2).
weight(P(i(k(k(i(x,x),k(x,y)),i(y,z)),i(x,z))),2).
weight(P(i(k(k(x,x),y),k(k(x,x),y))),2).
weight(P(i(k(k(x,x),y),k(x,k(x,y)))),2).
weight(P(i(k(n(x),n(x)),n(a(x,x)))),2).
weight(P(i(k(x,i(k(x,x),y)),i(x,y))),2).
weight(P(i(k(x,i(x,y)),y)),2).
weight(P(i(k(x,i(y,z)),i(y,k(x,z)))),2).
weight(P(i(k(x,k(x,y)),k(k(x,x),y))),2).
weight(P(i(k(x,n(x)),x)),2).
weight(P(i(k(x,y),k(y,x))),2).
weight(P(i(n(a(x,x)),k(n(x),n(x)))),2).
weight(P(i(n(n(n(x))),n(x))),2).
weight(P(i(n(x),i(n(x),n(a(x,x))))),2).
weight(P(i(n(x),n(n(n(x))))),2).
weight(P(i(x,i(n(x),y))),2).
weight(P(i(x,i(y,k(y,x)))),2).
weight(P(i(x,k(x,x))),2).
weight(P(i(x,n(n(x)))),2).
weight(P(i(x,x)),2).
weight(P(k(i(a(k(x,y),z),k(a(x,z),a(y,z))),i(k(a(x,z),a(y,z)),a(k(x,y),z)))),2).

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weight(P(k(i(i(x,i(y,z)),i(k(x,y),z)),i(i(k(x,y),z),i(x,i(y,z))))),2).
weight(P(k(i(i(x,i(y,z)),i(y,i(x,z))),i(i(y,i(x,z)),i(x,i(y,z))))),2).
weight(P(k(i(k(i(x,y),i(x,z)),i(x,k(y,z))),i(i(x,k(y,z)),k(i(x,y),i(x,z))))),2).
weight(P(k(i(n(a(x,y)),k(n(x),n(y))),i(k(n(x),n(y)),n(a(x,y))))),2).
weight(a(a(x,x),y),2).
weight(a(a(x,y),x),2).
weight(a(k(x,n(x)),x),2).
weight(a(k(x,x),y),2).
weight(a(k(x,y),k(x,y)),2).
weight(a(k(x,y),z),2).
weight(a(n(x),x),2).
weight(a(x,a(x,y)),2).
weight(a(x,n(x)),2).
weight(a(x,x),2).
weight(a(x,y),2).
weight(a(x,z),2).
weight(a(y,x),2).
weight(a(y,z),2).
weight(a(z,u),2).
weight(i(a(a(x,x),y),a(a(x,x),y)),2).
weight(i(a(a(x,x),y),a(a(x,y),x)),2).
weight(i(a(a(x,x),y),a(x,a(x,y))),2).
weight(i(a(k(x,n(x)),x),x),2).
weight(i(a(k(x,x),y),k(a(x,y),a(x,y))),2).
weight(i(a(k(x,y),k(x,y)),k(a(x,x),y)),2).
weight(i(a(k(x,y),z),k(a(x,z),a(y,z))),2).
weight(i(a(n(x),x),i(x,x)),2).
weight(i(a(x,a(x,y)),a(a(x,x),y)),2).
weight(i(a(x,n(x)),i(n(n(x)),x)),2).
weight(i(a(x,x),i(i(x,x),x)),2).
weight(i(a(x,x),i(n(x),x)),2).
weight(i(a(x,x),x),2).
weight(i(a(x,y),a(x,y)),2).
weight(i(a(x,y),a(x,z)),2).
weight(i(a(x,y),a(y,x)),2).
weight(i(a(x,y),z),2).
weight(i(i(a(x,x),x),i(x,x)),2).
weight(i(i(k(x,x),y),i(k(x,n(y)),n(x))),2).
weight(i(i(k(x,y),z),i(x,i(y,z))),2).
weight(i(i(x,i(y,z)),i(k(x,y),z)),2).
weight(i(i(x,i(y,z)),i(y,i(x,z))),2).
weight(i(i(x,k(y,z)),k(i(x,y),i(x,z))),2).
weight(i(i(x,n(x)),n(x)),2).
weight(i(i(x,x),i(a(x,x),x)),2).
weight(i(i(x,x),i(a(x,y),a(x,y))),2).
weight(i(i(x,x),i(i(x,y),i(x,y))),2).
weight(i(i(x,x),i(k(x,y),a(x,z))),2).
weight(i(i(x,x),i(n(n(x)),n(n(x)))),2).
weight(i(i(x,x),i(n(x),n(x))),2).
weight(i(i(x,x),i(x,i(y,x))),2).
weight(i(i(x,x),i(x,y)),2).
weight(i(i(x,x),x),2).
weight(i(i(x,y),i(i(x,x),i(x,y))),2).
```

weight(i(i(x,y),i(k(x,x),y)),2).
weight(i(i(x,y),i(k(x,z),k(y,z))),2).
weight(i(i(x,y),i(x,y)),2).
weight(i(i(x,y),i(x,z)),2).
weight(i(i(y,i(x,z)),i(x,i(y,z))),2).
weight(i(i(y,z),i(i(x,y),i(x,z))),2).
weight(i(k(a(x,x),n(x)),x),2).
weight(i(k(a(x,x),y),a(k(x,y),k(x,y))),2).
weight(i(k(a(x,y),a(x,y)),a(k(x,x),y)),2).
weight(i(k(a(x,z),a(y,z)),a(k(x,y),z)),2).
weight(i(k(i(x,x),i(y,z)),i(a(x,y),a(x,z))),2).
weight(i(k(i(x,x),n(x)),n(x)),2).
weight(i(k(i(x,y),i(x,n(y))),n(x)),2).
weight(i(k(i(x,y),i(x,z)),i(x,k(y,z))),2).
weight(i(k(i(x,y),i(y,z)),i(x,z)),2).
weight(i(k(i(x,y),i(z,z)),i(k(x,z),k(y,z))),2).
weight(i(k(i(x,z),i(y,z)),i(a(x,y),z)),2).
weight(i(k(k(i(x,a(x,y)),i(x,z)),i(y,u)),i(x,a(z,u))),2).
weight(i(k(k(i(x,a(x,y)),i(x,z)),i(y,z)),i(x,z)),2).
weight(i(k(k(i(x,x),k(x,y)),i(y,z)),i(x,z)),2).
weight(i(k(k(x,x),y),k(k(x,x),y)),2).
weight(i(k(k(x,x),y),k(x,k(x,y))),2).
weight(i(k(n(x),n(x)),n(a(x,x))),2).
weight(i(k(n(x),n(y)),n(a(x,y))),2).
weight(i(k(x,i(k(x,x),y)),i(x,y)),2).
weight(i(k(x,i(x,y)),y),2).
weight(i(k(x,i(y,z)),i(y,k(x,z))),2).
weight(i(k(x,k(x,y)),k(k(x,x),y)),2).
weight(i(k(x,n(x)),x),2).
weight(i(k(x,n(y)),n(x)),2).
weight(i(k(x,x),y),2).
weight(i(k(x,y),a(x,z)),2).
weight(i(k(x,y),k(y,x)),2).
weight(i(k(x,y),z),2).
weight(i(k(x,z),k(y,z)),2).
weight(i(n(a(x,x)),k(n(x),n(x))),2).
weight(i(n(a(x,y)),k(n(x),n(y))),2).
weight(i(n(n(n(x))),n(x)),2).
weight(i(n(n(x)),n(n(x))),2).
weight(i(n(n(x)),x),2).
weight(i(n(x),i(n(x),n(a(x,x)))),2).
weight(i(n(x),n(a(x,x))),2).
weight(i(n(x),n(n(n(x)))),2).
weight(i(n(x),n(x)),2).
weight(i(n(x),x),2).
weight(i(n(x),y),2).
weight(i(x,a(x,y)),2).
weight(i(x,a(z,u)),2).
weight(i(x,i(n(x),y)),2).
weight(i(x,i(y,k(y,x))),2).
weight(i(x,i(y,x)),2).
weight(i(x,i(y,z)),2).
weight(i(x,k(x,x)),2).

```
weight(i(x,k(y,z)),2).
weight(i(x,n(n(x))),2).
weight(i(x,n(x)),2).
weight(i(x,n(y)),2).
weight(i(x,x),2).
weight(i(x,y),2).
weight(i(x,z),2).
weight(i(y,i(x,z)),2).
weight(i(y,k(x,z)),2).
weight(i(y,k(y,x)),2).
weight(i(y,u),2).
weight(i(y,x),2).
weight(i(y,z),2).
weight(i(z,z),2).
weight(k(a(x,x),n(x)),2).
weight(k(a(x,x),y),2).
weight(k(a(x,y),a(x,y)),2).
weight(k(a(x,z),a(y,z)),2).
weight(k(i(a(k(x,y),z),k(a(x,z),a(y,z))),i(k(a(x,z),a(y,z)),a(k(x,y),z))),2).
weight(k(i(i(x,i(y,z)),i(k(x,y),z)),i(i(k(x,y),z),i(x,i(y,z)))),2).
weight(k(i(i(x,i(y,z)),i(y,i(x,z))),i(i(y,i(x,z)),i(x,i(y,z)))),2).
weight(k(i(k(i(x,y),i(x,z)),i(x,k(y,z))),i(i(x,k(y,z)),k(i(x,y),i(x,z)))),2).
weight(k(i(n(a(x,y)),k(n(x),n(y))),i(k(n(x),n(y)),n(a(x,y)))),2).
weight(k(i(x,a(x,y)),i(x,z)),2).
weight(k(i(x,x),i(y,z)),2).
weight(k(i(x,x),k(x,y)),2).
weight(k(i(x,x),n(x)),2).
weight(k(i(x,y),i(x,n(y))),2).
weight(k(i(x,y),i(x,z)),2).
weight(k(i(x,y),i(y,z)),2).
weight(k(i(x,y),i(z,z)),2).
weight(k(i(x,z),i(y,z)),2).
weight(k(k(i(x,a(x,y)),i(x,z)),i(y,u)),2).
weight(k(k(i(x,a(x,y)),i(x,z)),i(y,z)),2).
weight(k(k(i(x,x),k(x,y)),i(y,z)),2).
weight(k(k(x,x),y),2).
weight(k(n(x),n(x)),2).
weight(k(n(x),n(y)),2).
weight(k(x,i(k(x,x),y)),2).
weight(k(x,i(x,y)),2).
weight(k(x,i(y,z)),2).
weight(k(x,k(x,y)),2).
weight(k(x,n(x)),2).
weight(k(x,n(y)),2).
weight(k(x,x),2).
weight(k(x,y),2).
weight(k(x,z),2).
weight(k(y,x),2).
weight(k(y,z),2).
weight(n(a(x,x)),2).
weight(n(a(x,y)),2).
weight(n(n(n(x))),2).
weight(n(n(x)),2).
```

```
weight(n(x),2).
weight(junk,1000).
end_of_list.

list(usable).
% condensed detachment
-P(i(x,y)) | -P(x) | P(y).
%  P(i(k(x,y),y)).      % 4
P(i(i(x,y),i(i(x,z),i(x,k(y,z))))).   % 5
P(i(x,a(x,y))).           % 6
P(i(x,a(y,x))).           % 7
P(i(i(x,y),i(i(z,y),i(a(x,z),y)))). % 8
%  P(i(i(x,n(y)),i(y,n(x)))).    % 9
%  P(i(n(x),i(x,y))).       % 10
end_of_list.

list(sos).
%  Following 10 is an axiomatization of intuitionistic logic from Horn's 1962 paper
P(i(x,i(y,x))).      % 1
P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).   % 2
P(i(k(x,y),x)).      % 3
P(i(k(x,y),y)).      % 4
%  P(i(i(x,y),i(i(x,z),i(x,k(y,z))))).   % 5
%  P(i(x,a(x,y))).           % 6
%  P(i(x,a(y,x))).           % 7
%  P(i(i(x,y),i(i(z,y),i(a(x,z),y)))). % 8
%  P(i(i(x,n(y)),i(y,n(x)))).    % 9
%  P(i(n(x),i(x,y))).       % 10
end_of_list.

list(passive).
%  Following are eight of Heyting's axioms and four others.
-P(i(b,k(b,b))) | $ANS(01).
-P(i(k(b,c),k(c,b))) | $ANS(02).
-P(i(i(b,c),i(k(b,d),k(c,d)))) | $ANS(03).
-P(i(k(i(b,c),i(c,d)),i(b,d))) | $ANS(04).
-P(i(k(b,i(b,c)),c)) | $ANS(06).
-P(i(a(b,c),a(c,b))) | $ANS(08).
-P(i(k(i(b,d),i(c,d)),i(a(b,c),d))) | $ANS(09).
-P(i(k(i(b,c),i(b,n(c))),n(b))) | $ANS(11).
-P(i(i(q,r),i(i(p,q),i(p,r)))) | $ANS(H2).
-P(i(i(p,i(q,r)),i(q,i(p,r)))) | $ANS(H3).
-P(i(i(p,n(p)),n(p)))           | $ANS(N1).
-P(i(p,i(n(p),q))) | $ANS(N2).
%  Following theorems from Heyting's paper.
-P(i(p,p)) | $ANS(H221).
-P(i(k(i(p,q),i(r,d)),i(k(p,r),k(q,d)))) | $ANS(H223).
%  -PP(iff(k(i(p,q),i(p,r)),i(p,k(q,r)))) | $ANS(H224).
-P(i(k(p,i(q,r)),i(q,k(p,r)))) | $ANS(H225).
-P(i(p,i(q,k(q,p)))) | $ANS(H226).
%  -PP(iff(i(p,i(b,c)),i(k(p,b),c))) | $ANS(H227).
%  -PP(iff(i(p,i(b,c)),i(b,i(p,c)))) | $ANS(H2271).
-P(i(i(p,c),i(k(p,b),c))) | $ANS(H228).
```

```
-P(i(i(p,b),i(p,i(c,b)))) | $ANS(H2281).
-P(i(k(p,i(k(p,b),c)),i(b,c))) | $ANS(H2282).
-P(i(i(p,b),i(i(b,c),i(p,c)))) | $ANS(H229).
-P(i(i(b,c),i(i(p,b),i(p,c)))) | $ANS(H2291).
-P(i(k(k(p,b),c),k(p,k(b,c)))) | $ANS(H23).
-P(i(k(k(p,b),c),k(k(b,p),c))) | $ANS(H231).
-P(i(k(p,k(b,c)),k(k(p,b),c))) | $ANS(H232).
-P(i(k(k(i(p,b),k(b,c)),i(c,d)),i(p,d))) | $ANS(H24).
-P(i(a(a(p,b),c),a(p,a(b,c)))) | $ANS(H32).
-P(i(a(p,a(b,c)),a(a(p,b),c))) | $ANS(H321).
-P(i(a(p,p),p))              | $ANS(H322).
-P(i(k(i(p,b),i(c,d)),i(a(p,c),a(b,d)))) | $ANS(H33).
-P(i(i(p,b),i(k(p,c),a(b,d)))) | $ANS(H331).
-P(i(i(p,b),i(a(p,b),b)))     | $ANS(H332).
-P(i(i(a(p,b),b),i(p,b)))     | $ANS(H333).
-P(i(i(p,b),i(a(p,c),a(b,c)))) | $ANS(H334).
-P(i(a(a(p,b),c),a(a(b,p),c))) | $ANS(H335).
-P(i(a(a(b,p),c),a(a(p,c),b))) | $ANS(H3351).
-P(i(a(k(p,c),k(b,c)),k(a(p,b),c))) | $ANS(H34).
-P(i(k(a(p,b),c),a(k(p,c),k(b,c)))) | $ANS(H341).
-P(i(a(k(p,b),c),k(a(p,c),a(b,c)))) | $ANS(H3421).
-P(i(k(a(p,c),a(b,c)),a(k(p,b),c))) | $ANS(H3422).
%  -PP(iff(a(k(p,b),c),k(a(p,c),a(b,c)))) | $ANS(H342).
-P(i(k(k(i(p,a(b,c)),i(b,d)),i(c,e)),i(p,a(d,e)))) | $ANS(H35).
-P(i(k(k(i(p,a(b,c)),i(b,d)),i(c,d)),i(p,d))) | $ANS(H351).
-P(i(a(p,b),i(i(p,b),b)))  | $ANS(H36).
-P(i(i(p,b),i(n(b),n(p)))) | $ANS(H42).
-P(i(i(p,b),i(n(n(p)),n(n(b))))) | $ANS(H422).
-P(i(i(k(p,b),c),i(k(p,n(c)),n(b)))) | $ANS(H423).
-P(i(k(i(p,b),n(b)),n(p))) | $ANS(H424).
-P(i(p,n(n(p)))) | $ANS(H43).
-P(i(n(p),n(n(n(p))))) | $ANS(H431).
-P(i(n(n(n(p))),n(p))) | $ANS(H432).
-P(i(k(p,n(p)),b)) | $ANS(H44).
-P(i(a(k(p,n(p)),b),b)) | $ANS(H441).
-P(i(k(a(p,b),n(p)),b)) | $ANS(H442).
-P(i(n(p),i(n(b),n(a(p,b))))) | $ANS(H443).
-P(i(n(a(p,b)),k(n(p),n(b)))) | $ANS(H4441).
-P(i(k(n(p),n(b)),n(a(p,b)))) | $ANS(H4442).
%  -PP(iff(n(a(p,b)),k(n(p),n(b)))) | $ANS(H444).
-P(i(a(p,n(p)),i(n(n(p)),p))) | $ANS(H445).
-P(i(a(n(p),b),i(p,b))) | $ANS(H446).
-P(i(a(p,b),i(n(p),b))) | $ANS(H447).
%  Following 5 are expanded theorems corresponding to iff theorems.
-P(k(i(k(i(p,q),i(p,r)),i(p,k(q,r))),i(i(p,k(q,r)),k(i(p,q),i(p,r))))) | $ANS(HIF1).
-P(k(i(i(p,i(q,r)),i(k(p,q),r)),i(i(k(p,q),r),i(p,i(q,r))))) | $ANS(HIF2).
-P(k(i(i(p,i(q,r)),i(q,i(p,r))),i(i(q,i(p,r)),i(p,i(q,r))))) | $ANS(HIF3).
-P(k(i(a(k(p,q),r),k(a(p,r),a(q,r))),i(k(a(p,r),a(q,r)),a(k(p,q),r)))) | $ANS(HIF4).
-P(k(i(n(a(p,q)),k(n(p),n(q))),i(k(n(p),n(q)),n(a(p,q))))) | $ANS(HIF5).
end_of_list.

list(demodulators).
n(n(x)) = junk.
```

```
i(x,junk) = junk.
i(junk,x) = junk.
n(junk) = junk.
end_of_list.
```

3. Traveling in Earnest, First Approach

The beginning of the journey, in focus in the preceding section, was indeed fruitful. In the corresponding experiment, OTTER proved (I believe) twenty-six of the sixty-three. Those results I immediately used in an action that I refer to as *lemma adjunction*, in its fuller form; see Section 4 for a detailed discussion. With lemma adjunction, you include in the so-called next run the lemmas of interest proved in the so-ca;led preceding run. In the fuller form, you include, in addition to the lemmas, the proof steps of the newly proved lemmas. The cited proof steps are placed in list(sos) to cause the program to key on them quickly as it attempts to make further progress. the command, set(sos_first), has the program focus, for inference-rule application, on members of the initial set of support before focusing on newly deduced clauses. Lemma adjunction, in its strict use or more fuller use, can be viewed as addressing the program's lack of knowledge by adding, as you travel, to its initial bit of knowledge.

Specifically, I adjoined ninety-nine formulas (which can be thought of as lemmas) to list(sos) in the second experiment. Those formulas came from sorting (to remove duplicates) the proof steps in the proofs of the twenty-six proved in the first experiment. The second input file, when compared with the first, evinced one additional change. In particular, I (so-to-speak) moved Horn's fifth axiom from the usable list to the (initial) set of support list to enable OTTER to use it to initiate lines of reasoning. In answer to the pertinent question that (in effect) asks why the fifth axiom was not already in list(sos) in the first experiment, I guess I was not as consistent as I could have been. After all, the fourth and fifth axioms each focus on the function k.

The presence in the input of the proof steps from the first experiment, of course, caused the program to avoid supplying proofs of the corresponding theorems. Although I allowed a fair amount of CPU time to be utilized, only three new theorems were proved: H3421, H223, and H23. I was still relying for directing the reasoning on the cited two sets of subformulas.

The third experiment used an input file like the second but with two important changes. First, I decided to give the program permission to consider Horn's ninth and tenth axioms, those concerned with negation. After all, why keep OTTER in the dark in this context. Second, continuing in the spirit of full lemma adjunction, I adjoined to list(sos) seventeen sorted proof steps obtained from the three new proofs.

The third experiment proved eight additional theorems. Not very surprising, each of the eight involved the function n for negation, the following in order: N2, H446, H447, H445, N1, H44, H441, and H442.

The next two experiments continued the same pattern of attack, each proving additional theorems from among the sixty-three. The first of the two proved three of the sixty-three, and the second proved two. At this point in my journey, if memory serves, I paused to see what might occur, with a sixth experiment, if I tried a different approach; I was also curious about where things stood in my attempt to prove all sixty-three. (In Input File 2, given in the next section, you will be shown the spirit of what occurred in this sixth experiment.) In particular, I placed in the initial set of support Horn's axioms 1, 2, 3, 9, and 10, using the remaining five only to complete applications of condensed detachment. I replaced the breadth-first (level-saturation) search with a search based on the complexity of formulas, relying on McCune's ratio strategy, with an assignment of the value 2 to pick_given_ratio. With that value, the program chooses 2 clauses (for inference-rule initiation) by complexity, 1 by first come first serve, then 2, 1, and the like. I made one additional important change. Specifically, I ceased relying on subformulas, as I had in the previous five experiments. Instead, I included (after sorting to remove duplicates) proof steps of the theorems (from among the sixty-three) that OTTER had proved in the first five runs.

This radically different approach (from the first five experiments) returned forty-seven proofs from the sought-after sixty-three. Approximately 40 CPU-minutes sufficed. But now, because of my tendency to conduct numerous experiments more or less at the same time, the precise path I took is a bit hazy. You see, rather than waiting for each experiment to complete, I often begin a new one, sometimes borrowing from the partial results so far obtained. Also, I have not chosen to detail runs that yielded little of interest, preferring instead to capture the essence of the approach that eventually succeeded.

For the seventh experiment, I returned to the fuller lemma-adjunction approach, including in the initial set of support proof steps from proofs found in the first five experiments. From among the Horn axioms, I placed in the initial list(sos) 1, 2, 3, 4, 9, and 10. As in the first five experiments, I relied (for directing the reasoning) on the two sets of subformulas already discussed. However, I did borrow from the run that showed me where things stood, namely, by using a search based on complexity (rather than on breadth first). I assigned the value 8 to max_weight, the value 5 to max_distinct_vars, and the value 4 to pick_given_ratio. After 200 formulas were chosen to drive the reasoning, I instructed OTTER to change the value of max_weight to 7. My actions were designed to enable the program to explore far deeper into the search space of deducible conclusions. The program found seven new proofs, the last of which required more than 4 CPU-hours. Yes, things were getting tougher.

4. A Radically Different Second Approach

Now, to conduct an eighth experiment, I turned away from lemma adjunction, as you will see in the promised second input file to be given shortly, a file that captures the essence of the sixth experiment. Indeed, I thought that something like the sixth experiment merited imitation. In particular, for the next lengthy part of the journey, I turned to a complexity-directed search, rather than a breadth-first search. Lemma adjunction was now not the choice; instead, I relied on *resonators*, formulas (or equations) that are used to direct the program's reasoning, where the variables are treated as indistinguishable. The formulas I included, as you see, corresponded to proof steps of proofs of theorems already proved from among the sixty-three targets. They are placed in weight_list(pick_and_purge). The adjunction or inclusion of a resonator is distinctly different from the inclusion or adjunction of a lemma (or proof step) in that resonators do not take on **true** or **false** values, in contrast to lemmas that, when included, are assumed **true**. As is the case in a course, a paper, or a book, the introduction of a lemma usually implies it will soon be used and its use will materially aid the reaching of a goal, for example, the proof of some theorem in focus. In contrast, the introduction of a resonator suggests that its pattern of symbols, where all variables are treated as just variables indistinguishable from each other, is conjectured to point the way to reaching the target. The smaller the value assigned to a resonator, the stronger the conjecture is that items matching the resonator merit heavy focus for inference-rule initiation.

Input File 2

```
set(hyper_res).
assign(max_weight,15).
% assign(change_limit_after,200).
% assign(new_max_weight,6).
clear(print_kept).
% set(process_input).
% set(ancestor_subsume).
set(back_sub).
clear(print_back_sub).
assign(max_distinct_vars,6).
assign(pick_given_ratio,2).
assign(max_mem,1900000).
% assign(max_seconds,1).
% assign(max_given,24).
assign(max_proofs,-1).
assign(report,5400).
set(order_history).
```

```
set(input_sos_first).
% set(sos_queue).

weight_list(pick_and_purge).
% Following 99 sorted proof steps of proofs in temp.beeson.heyting.out9a, in which 26 of 63 were proved.
weight(P(i(a(a(x,y),z),a(a(y,x),z))),1).
weight(P(i(a(k(x,y),k(z,y)),k(a(x,z),y))),1).
weight(P(i(a(x,x),i(y,x))),1).
weight(P(i(a(x,x),x)),1).
weight(P(i(a(x,y),a(y,x))),1).
weight(P(i(a(x,y),i(i(x,y),y))),1).
weight(P(i(i(a(x,y),z),i(x,z))),1).
weight(P(i(i(i(x,i(y,z)),i(x,y)),i(i(x,i(y,z)),i(x,z)))),1).
weight(P(i(i(i(x,y),a(x,y)),i(i(x,y),y))),1).
weight(P(i(i(i(x,y),i(x,z)),i(i(x,y),i(x,k(y,z))))),1).
weight(P(i(i(i(x,y),i(z,y)),i(i(i(x,y),a(x,z)),i(i(x,y),y)))),1).
weight(P(i(i(i(x,y),i(z,y)),i(i(x,y),i(a(x,z),y)))),1).
weight(P(i(i(i(x,y),z),i(y,z))),1).
weight(P(i(i(k(x,i(y,z)),y),i(k(x,i(y,z)),z))),1).
weight(P(i(i(k(x,y),i(x,z)),i(k(x,y),z))),1).
weight(P(i(i(k(x,y),i(y,z)),i(k(x,y),z))),1).
weight(P(i(i(k(x,y),k(z,u)),i(k(x,y),k(u,y)))),1).
weight(P(i(i(k(x,y),z),i(i(u,k(z,y)),i(a(k(x,y),u),k(z,y))))),1).
weight(P(i(i(k(x,y),z),i(k(x,y),k(x,z)))),1).
weight(P(i(i(k(x,y),z),i(k(x,y),k(y,z)))),1).
weight(P(i(i(k(x,y),z),i(k(x,y),k(z,y)))),1).
weight(P(i(i(k(x,y),z),i(u,i(k(x,y),k(y,z))))),1).
weight(P(i(i(x,a(y,z)),i(a(x,y),a(y,z)))),1).
weight(P(i(i(x,i(a(x,y),z)),i(x,z))),1).
weight(P(i(i(x,i(i(k(y,z),y),u)),i(x,u))),1).
weight(P(i(i(x,i(i(k(y,z),z),u)),i(x,u))),1).
weight(P(i(i(x,i(i(y,a(z,y)),u)),i(x,u))),1).
weight(P(i(i(x,i(i(y,i(z,y)),u)),i(x,u))),1).
weight(P(i(i(x,i(i(y,x),z)),i(x,z))),1).
weight(P(i(i(x,i(k(y,i(z,u)),z)),i(x,i(k(y,i(z,u)),u)))),1).
weight(P(i(i(x,i(k(y,z),u)),i(x,i(k(y,z),k(u,z))))),1).
weight(P(i(i(x,i(y,i(z,u))),i(x,i(i(y,z),i(y,u))))),1).
weight(P(i(i(x,i(y,z)),i(a(z,x),i(y,z)))),1).
weight(P(i(i(x,i(y,z)),i(i(u,i(x,y)),i(u,i(x,z))))),1).
weight(P(i(i(x,i(y,z)),i(u,i(i(x,y),i(x,z))))),1).
weight(P(i(i(x,i(y,z)),i(x,i(i(u,z),i(a(y,u),z))))),1).
weight(P(i(i(x,i(y,z)),i(x,i(i(y,u),i(y,k(z,u)))))),1).
weight(P(i(i(x,i(y,z)),i(y,i(x,z)))),1).
weight(P(i(i(x,k(a(y,z),u)),i(a(k(y,u),x),k(a(y,z),u)))),1).
weight(P(i(i(x,k(y,z)),i(i(x,u),i(x,k(z,u))))),1).
weight(P(i(i(x,k(y,z)),i(x,z))),1).
weight(P(i(i(x,y),i(a(x,y),y))),1).
weight(P(i(i(x,y),i(a(x,z),a(y,z)))),1).
weight(P(i(i(x,y),i(i(y,z),i(x,z)))),1).
weight(P(i(i(x,y),i(i(z,a(y,u)),i(a(x,z),a(y,u))))),1).
weight(P(i(i(x,y),i(i(z,x),i(z,a(y,u))))),1).
weight(P(i(i(x,y),i(i(z,x),i(z,y)))),1).
weight(P(i(i(x,y),i(k(x,z),a(y,u)))),1).
```

```
weight(P(i(i(x,y),i(k(x,z),k(y,z)))),1).
weight(P(i(i(x,y),i(k(x,z),y))),1).
weight(P(i(i(x,y),i(x,a(y,z)))),1).
weight(P(i(i(x,y),i(x,a(z,y)))),1).
weight(P(i(i(x,y),i(x,i(z,y)))),1).
weight(P(i(i(x,y),i(x,k(y,x)))),1).
weight(P(i(i(x,y),i(x,x))),1).
weight(P(i(i(x,y),i(z,i(i(x,u),i(x,k(y,u)))))),1).
weight(P(i(i(x,y),i(z,i(x,a(y,u))))),1).
weight(P(i(k(i(x,y),i(y,z)),i(x,z))),1).
weight(P(i(k(i(x,y),i(z,y)),i(a(x,z),y))),1).
weight(P(i(k(i(x,y),z),i(i(u,y),i(a(x,u),y)))),1).
weight(P(i(k(k(x,y),z),k(k(y,x),z))),1).
weight(P(i(k(k(x,y),z),k(y,k(y,x)))),1).
weight(P(i(k(x,i(k(x,y),z)),i(y,z))),1).
weight(P(i(k(x,i(x,y)),y)),1).
weight(P(i(k(x,i(y,z)),i(i(u,y),i(u,z)))),1).
weight(P(i(k(x,i(y,z)),i(y,k(x,z)))),1).
weight(P(i(k(x,y),a(x,z))),1).
weight(P(i(k(x,y),a(z,x))),1).
weight(P(i(k(x,y),i(i(z,u),i(z,k(x,u))))),1).
weight(P(i(k(x,y),i(z,x))),1).
weight(P(i(k(x,y),i(z,y))),1).
weight(P(i(k(x,y),k(a(z,x),y))),1).
weight(P(i(k(x,y),k(y,x))),1).
weight(P(i(x,i(i(i(y,z),i(u,y)),i(i(y,z),i(u,z))))),1).
weight(P(i(x,i(i(k(y,i(z,u)),z),i(k(y,i(z,u)),u)))),1).
weight(P(i(x,i(i(k(y,z),u),i(k(y,z),k(y,u))))),1).
weight(P(i(x,i(i(y,i(a(y,z),u)),i(y,u)))),1).
weight(P(i(x,i(i(y,i(i(z,y),u)),i(y,u)))),1).
weight(P(i(x,i(i(y,i(z,u)),i(i(y,z),i(y,u))))),1).
weight(P(i(x,i(i(y,z),i(i(u,z),i(a(y,u),z))))),1).
weight(P(i(x,i(i(y,z),i(i(y,u),i(y,k(z,u)))))),1).
weight(P(i(x,i(i(y,z),i(y,k(y,z))))),1).
weight(P(i(x,i(i(y,z),i(y,y)))),1).
weight(P(i(x,i(k(y,i(k(y,x),z)),z))),1).
weight(P(i(x,i(k(y,z),k(y,x)))),1).
weight(P(i(x,i(k(y,z),k(z,k(z,y))))),1).
weight(P(i(x,i(k(y,z),y))),1).
weight(P(i(x,i(k(y,z),z))),1).
weight(P(i(x,i(y,a(y,z)))),1).
weight(P(i(x,i(y,a(z,y)))),1).
weight(P(i(x,i(y,i(k(z,u),u)))),1).
weight(P(i(x,i(y,i(k(z,u),z)))),1).
weight(P(i(x,i(y,i(z,a(u,z))))),1).
weight(P(i(x,i(y,i(z,i(u,z))))),1).
weight(P(i(x,i(y,i(z,y)))),1).
weight(P(i(x,i(y,k(y,x)))),1).
weight(P(i(x,i(y,y))),1).
weight(P(i(x,k(x,x))),1).
weight(P(i(x,x)),1).
% Following 17 are sorted proof steps of 3 proofs, .out9b.
weight(P(i(a(k(x,y),z),a(y,z))),1).
```

weight(P(i(a(k(x,y),z),k(a(x,z),a(y,z)))),1).
weight(P(i(i(a(k(x,y),z),u),i(a(k(x,y),z),k(a(x,z),u)))),1).
weight(P(i(i(i(k(x,k(y,z)),z),u),u)),1).
weight(P(i(i(k(x,k(y,z)),u),i(k(x,k(y,z)),k(k(x,y),u)))),1).
weight(P(i(i(k(x,y),i(y,z)),i(i(k(x,y),u),i(k(x,y),k(z,u))))),1).
weight(P(i(i(k(x,y),i(z,u)),i(k(x,y),i(z,k(x,u))))),1).
weight(P(i(i(x,i(k(x,y),z)),i(k(x,y),z))),1).
weight(P(i(i(x,i(y,z)),i(i(y,x),i(y,z)))),1).
weight(P(i(i(x,y),i(i(a(x,z),u),i(a(x,z),k(a(y,z),u))))),1).
weight(P(i(k(k(x,y),z),k(x,k(y,z)))),1).
weight(P(i(k(x,k(y,z)),k(k(x,y),z))),1).
weight(P(i(k(x,k(y,z)),z)),1).
weight(P(i(k(x,y),i(k(z,u),k(x,k(y,u))))),1).
weight(P(i(k(x,y),i(k(z,u),k(x,z)))),1).
weight(P(i(k(x,y),i(k(z,u),k(y,u)))),1).
weight(P(i(x,i(i(x,y),y))),1).
% Following 21 sorted proof steps of 8 proofs, .out9c.
weight(P(i(a(k(x,n(x)),y),y)),1).
weight(P(i(a(n(x),y),i(x,y))),1).
weight(P(i(a(x,y),i(n(x),y))),1).
weight(P(i(a(x,y),i(n(y),x))),1).
weight(P(i(i(i(k(x,y),x),i(z,u)),i(a(z,u),u))),1).
weight(P(i(i(i(k(x,y),z),u),i(i(x,z),u))),1).
weight(P(i(i(k(x,n(y)),y),i(k(x,n(y)),z))),1).
weight(P(i(i(k(x,y),i(y,i(z,u))),i(i(k(x,y),z),i(k(x,y),u)))),1).
weight(P(i(i(x,i(y,n(x))),i(x,n(y)))),1).
weight(P(i(i(x,i(y,z)),i(a(n(y),x),i(y,z)))),1).
weight(P(i(i(x,i(y,z)),i(a(x,z),i(y,z)))),1).
weight(P(i(i(x,i(y,z)),i(k(x,y),z))),1).
weight(P(i(i(x,i(y,z)),i(x,i(a(y,z),z)))),1).
weight(P(i(i(x,n(x)),i(x,y))),1).
weight(P(i(i(x,n(x)),n(x))),1).
weight(P(i(k(a(x,y),n(x)),y)),1).
weight(P(i(k(x,n(x)),y)),1).
weight(P(i(k(x,y),n(k(z,n(x))))),1).
weight(P(i(x,i(n(x),y))),1).
weight(P(i(x,i(n(y),i(y,z)))),1).
weight(P(i(x,n(k(y,n(x))))),1).
% Following 11 sorted proof steps, proving 3 more, .out9d.
weight(P(i(i(i(i(x,a(y,z)),i(a(u,x),a(y,z))),v),i(i(u,y),v))),1).
weight(P(i(i(i(i(x,y),i(z,y)),u),i(i(z,x),u))),1).
weight(P(i(i(i(x,a(y,z)),u),i(i(x,z),u))),1).
weight(P(i(i(i(x,y),z),i(n(x),z))),1).
weight(P(i(i(x,i(y,i(z,n(y)))),i(x,i(y,n(z))))),1).
weight(P(i(i(x,y),i(i(z,u),i(a(x,z),a(y,u))))),1).
weight(P(i(i(x,y),i(n(y),i(x,z)))),1).
weight(P(i(i(x,y),i(n(y),n(x)))),1).
weight(P(i(k(i(x,y),i(z,u)),i(a(x,z),a(y,u)))),1).
weight(P(i(x,k(x,i(i(y,i(z,u)),i(z,i(y,u)))))),1).
weight(P(k(i(i(x,i(y,z)),i(y,i(x,z))),i(i(u,i(v,w)),i(v,i(u,w))))),1).
% Following 4 prove two more theorems, .out9e.
weight(P(i(i(x,i(y,z)),i(x,i(n(z),n(y))))),1).
weight(P(i(k(i(x,y),n(y)),n(x))),1).

```
weight(P(i(n(x),i(a(x,y),y))),1).
weight(P(i(n(x),i(n(y),n(a(x,y))))),1).
% Following sorted 30 proof steps prove 6 more, .out9h.
weight(P(i(a(a(x,y),z),a(a(y,z),x))),1).
weight(P(i(a(a(x,y),z),a(x,a(y,z)))),1).
weight(P(i(a(x,a(y,z)),a(a(x,y),z))),1).
weight(P(i(a(x,a(y,z)),a(a(z,x),y))),1).
weight(P(i(a(x,y),a(a(x,z),y))),1).
weight(P(i(a(x,y),a(a(y,z),x))),1).
weight(P(i(a(x,y),a(a(z,x),y))),1).
weight(P(i(i(a(x,y),z),i(a(y,x),z))),1).
weight(P(i(i(i(i(x,y),i(x,k(z,y))),u),i(i(x,z),u))),1).
weight(P(i(i(i(k(x,y),z),u),i(i(y,z),u))),1).
weight(P(i(i(i(x,a(x,y)),z),z)),1).
weight(P(i(i(i(x,a(y,x)),z),z)),1).
weight(P(i(i(i(x,k(y,z)),u),i(k(y,i(x,z)),u))),1).
weight(P(i(i(k(x,y),z),i(i(x,u),i(k(x,y),k(z,u))))),1).
weight(P(i(i(x,a(y,z)),i(a(x,a(u,y)),a(a(y,z),u)))),1).
weight(P(i(i(x,a(y,z)),i(a(x,a(z,u)),a(a(y,z),u)))),1).
weight(P(i(i(x,a(y,z)),i(i(u,y),i(a(u,x),a(y,z))))),1).
weight(P(i(i(x,a(y,z)),i(x,a(z,y)))),1).
weight(P(i(i(x,i(y,k(z,u))),i(x,i(y,k(u,z))))),1).
weight(P(i(i(x,i(y,n(y))),i(x,n(y)))),1).
weight(P(i(i(x,i(y,z)),i(k(y,x),z))),1).
weight(P(i(i(x,k(i(y,z),n(z))),i(x,n(y)))),1).
weight(P(i(i(x,y),i(i(z,u),i(k(z,x),k(y,u))))),1).
weight(P(i(i(x,y),i(k(z,x),y))),1).
weight(P(i(k(i(x,y),i(x,n(y))),n(x))),1).
weight(P(i(k(i(x,y),i(z,n(y))),i(z,n(x)))),1).
weight(P(i(k(i(x,y),i(z,u)),i(k(x,z),k(u,y)))),1).
weight(P(i(k(i(x,y),i(z,u)),i(k(x,z),k(y,u)))),1).
weight(P(i(k(n(x),n(y)),n(a(x,y)))),1).
weight(P(i(x,i(k(y,z),k(z,y)))),1).
weight(junk,1000).
end_of_list.

list(usable).
% condensed detachment
-P(i(x,y)) | -P(x) | P(y).
P(i(k(x,y),y)).     % 4
P(i(i(x,y),i(i(x,z),i(x,k(y,z))))).   % 5
P(i(x,a(x,y))).           % 6
P(i(x,a(y,x))).           % 7
P(i(i(x,y),i(i(z,y),i(a(x,z),y)))). % 8
%  P(i(i(x,n(y)),i(y,n(x)))).   % 9
%  P(i(n(x),i(x,y))).      % 10
end_of_list.

list(sos).
% Following 10 is an axiomatization of intuitionistic logic from Horn's 1962 paper
P(i(x,i(y,x))).     % 1
P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).   % 2
P(i(k(x,y),x)).     % 3
```

```
% P(i(k(x,y),y)).     % 4
% P(i(i(x,y),i(i(x,z),i(x,k(y,z))))).  % 5
% P(i(x,a(x,y))).          % 6
% P(i(x,a(y,x))).          % 7
% P(i(i(x,y),i(i(z,y),i(a(x,z),y)))). % 8
P(i(i(x,n(y)),i(y,n(x)))).   % 9
P(i(n(x),i(x,y))).      % 10
end_of_list.

list(passive).
% Following are eight of Heyting's axioms and four others.
-P(i(b,k(b,b))) | $ANS(01).
-P(i(k(b,c),k(c,b))) | $ANS(02).
-P(i(i(b,c),i(k(b,d),k(c,d)))) | $ANS(03).
-P(i(k(i(b,c),i(c,d)),i(b,d))) | $ANS(04).
-P(i(k(b,i(b,c)),c)) | $ANS(06).
-P(i(a(b,c),a(c,b))) | $ANS(08).
-P(i(k(i(b,d),i(c,d)),i(a(b,c),d))) | $ANS(09).
-P(i(k(i(b,c),i(b,n(c))),n(b))) | $ANS(11).
-P(i(i(q,r),i(i(p,q),i(p,r)))) | $ANS(H2).
-P(i(i(p,i(q,r)),i(q,i(p,r)))) | $ANS(H3).
-P(i(i(p,n(p)),n(p)))          | $ANS(N1).
-P(i(p,i(n(p),q))) | $ANS(N2).
% Following theorems from Heyting's paper.
-P(i(p,p)) | $ANS(H221).
-P(i(k(i(p,q),i(r,d)),i(k(p,r),k(q,d)))) | $ANS(H223).
% -P(iff(k(i(p,q),i(p,r)),i(p,k(q,r)))) | $ANS(H224).
-P(i(k(p,i(q,r)),i(q,k(p,r)))) | $ANS(H225).
-P(i(p,i(q,k(q,p)))) | $ANS(H226).
% -P(iff(i(p,i(b,c)),i(k(p,b),c))) | $ANS(H227).
% -P(iff(i(p,i(b,c)),i(b,i(p,c)))) | $ANS(H2271).
-P(i(i(p,c),i(k(p,b),c))) | $ANS(H228).
-P(i(i(p,b),i(p,i(c,b)))) | $ANS(H2281).
-P(i(k(p,i(k(p,b),c)),i(b,c))) | $ANS(H2282).
-P(i(i(p,b),i(i(b,c),i(p,c)))) | $ANS(H229).
-P(i(i(b,c),i(i(p,b),i(p,c)))) | $ANS(H2291).
-P(i(k(k(p,b),c),k(p,k(b,c)))) | $ANS(H23).
-P(i(k(k(p,b),c),k(k(b,p),c))) | $ANS(H231).
-P(i(k(p,k(b,c)),k(k(p,b),c))) | $ANS(H232).
-P(i(k(k(i(p,b),k(b,c)),i(c,d)),i(p,d))) | $ANS(H24).
-P(i(a(a(p,b),c),a(p,a(b,c)))) | $ANS(H32).
-P(i(a(p,a(b,c)),a(a(p,b),c))) | $ANS(H321).
-P(i(a(p,p),p))             | $ANS(H322).
-P(i(k(i(p,b),i(c,d)),i(a(p,c),a(b,d)))) | $ANS(H33).
-P(i(i(p,b),i(k(p,c),a(b,d)))) | $ANS(H331).
-P(i(i(p,b),i(a(p,b),b)))    | $ANS(H332).
-P(i(i(a(p,b),b),i(p,b)))    | $ANS(H333).
-P(i(i(p,b),i(a(p,c),a(b,c)))) | $ANS(H334).
-P(i(a(a(p,b),c),a(a(b,p),c))) | $ANS(H335).
-P(i(a(a(b,p),c),a(a(p,c),b))) | $ANS(H3351).
-P(i(a(k(p,c),k(b,c)),k(a(p,b),c))) | $ANS(H34).
-P(i(k(a(p,b),c),a(k(p,c),k(b,c)))) | $ANS(H341).
-P(i(a(k(p,b),c),k(a(p,c),a(b,c)))) | $ANS(H3421).
```

```
-P(i(k(a(p,c),a(b,c)),a(k(p,b),c))) | $ANS(H3422).
% -P(iff(a(k(p,b),c),k(a(p,c),a(b,c)))) | $ANS(H342).
-P(i(k(k(i(p,a(b,c)),i(b,d)),i(c,e)),i(p,a(d,e)))) | $ANS(H35).
-P(i(k(k(i(p,a(b,c)),i(b,d)),i(c,d)),i(p,d))) | $ANS(H351).
-P(i(a(p,b),i(i(p,b),b)))  | $ANS(H36).
-P(i(i(p,b),i(n(b),n(p)))) | $ANS(H42).
-P(i(i(p,b),i(n(n(p)),n(n(b))))) | $ANS(H422).
-P(i(i(k(p,b),c),i(k(p,n(c)),n(b)))) | $ANS(H423).
-P(i(k(i(p,b),n(b)),n(p))) | $ANS(H424).
-P(i(p,n(n(p)))) | $ANS(H43).
-P(i(n(p),n(n(n(p))))) | $ANS(H431).
-P(i(n(n(n(p))),n(p))) | $ANS(H432).
-P(i(k(p,n(p)),b)) | $ANS(H44).
-P(i(a(k(p,n(p)),b),b)) | $ANS(H441).
-P(i(k(a(p,b),n(p)),b)) | $ANS(H442).
-P(i(n(p),i(n(b),n(a(p,b))))) | $ANS(H443).
-P(i(n(a(p,b)),k(n(p),n(b)))) | $ANS(H4441).
-P(i(k(n(p),n(b)),n(a(p,b)))) | $ANS(H4442).
% -P(iff(n(a(p,b)),k(n(p),n(b)))) | $ANS(H444).
-P(i(a(p,n(p)),i(n(n(p)),p))) | $ANS(H445).
-P(i(a(n(p),b),i(p,b))) | $ANS(H446).
-P(i(a(p,b),i(n(p),b))) | $ANS(H447).
% Following 5 are expanded theorems corresponding to iff theorems.
-P(k(i(k(i(p,q),i(p,r)),i(p,k(q,r))),i(i(p,k(q,r)),k(i(p,q),i(p,r))))) | $ANS(HIF1).
-P(k(i(i(p,i(q,r)),i(k(p,q),r)),i(i(k(p,q),r),i(p,i(q,r))))) | $ANS(HIF2).
-P(k(i(i(p,i(q,r)),i(q,i(p,r))),i(i(q,i(p,r)),i(p,i(q,r))))) | $ANS(HIF3).
-P(k(i(a(k(p,q),r),k(a(p,r),a(q,r))),i(k(a(p,r),a(q,r)),a(k(p,q),r)))) | $ANS(HIF4).
-P(k(i(n(a(p,q)),k(n(p),n(q))),i(k(n(p),n(q)),n(a(p,q))))) | $ANS(HIF5).
end_of_list.

list(demodulators).
n(n(n(x))) = junk.
iff(x,y) = k(i(x,y),i(y,x)).
% n(n(x)) = junk.
i(x,junk) = junk.
i(junk,x) = junk.
n(junk) = junk.
end_of_list.
```

In order at this time is a focus on some of the aspects of using lemma adjunction versus the use of resonators. With the former, as noted, the idea is to supplement, augment, the initial problem description, to adjoin newly proved lemmas and, possibly, the proof steps of the corresponding proofs. Typically, in the beginning, you place in list(passive) the negated form of lemmas or theorems you are attempting to prove. When, for one reason or another, an experiment (run) is terminated with one or more proofs of one or more targets, in the next run the newly proved results possibly coupled with the corresponding proof steps are adjoined to the (initial) list(sos). You might say that the program is learning about the field of interest. You also usually include the command set(input_sos_first) to cause the program to focus on the items in list(sos) before focusing on items deduced during the run. When items in the now amended set of support are used in yet even newer proofs, as you realize, those newer proofs are not based strictly on the original problem description. Therefore, when you are finished, if your goal is to supply proofs of various theorems of interest based solely on the original problem description, the Horn axioms in the case featured in this notebook, you must take some action to produce proofs in which the adjoined items are not present. Despite this possibly apparent drawback, the progress you can make by relying on lemma adjunction often leads to proofs of concern that can, in turn, be used to reach the major goal, that of proofs based solely (here) on Horn's ten axioms.

With the resonators, when and if your program produces proofs of targets of interest, the only items used are those from the initial problem description. Now, it merits mention, the resonators that you use need not correspond to results that hold in the theory or field under study. Each resonator merely provides a pattern conjectured to be of use in directing, as a strategy, the reasoning of the program. Lemma adjunction, when compared with the use of resonators, often increases the likelihood of finding proofs by (in effect) mirroring the actions of a published paper or book. Many sources exist for resonators, for example, other areas of logic or mathematics, or formulas or equations that you suspect have promising functional patterns, or simply wild guesses. In the case of Input File 2, the resonators were (as you see) taken from theorems, from among the sixty-three, proved in earlier runs. So, in my research, I do rely on lemma adjunction often and, far more often, on the use of resonators.

In Input File 2, I assigned the value of 2 to pick_given_ratio to instruct OTTER to choose, for inference-rule initiation, 2 clauses by complexity, 1 by first come first serve, then 2, then 2, and so on. My choice was motivated by the thought that quite complex newly deduced items might be needed, which meant the program must not emphasize highly the choice of focus based on complexity. I assigned the value 6 to max_distinct_vars, suspecting that some of the proofs that were sought might depend on deduced formulas relying on six distinct variables. With an assignment of -1 to max_proofs, the program was told to find as many proofs as it could within memory constraints, 190000 kilobytes. I was aware that the assigned value of 15 to max_weight might be too small, that, indeed, some of the proofs might require one or more deduced formulas relying on more than fifteen symbols (not counting, of course, commas and parentheses). Nevertheless, I was concerned that a much larger value would cause the program to drown in too many newly deduced items. The use of Input File 2 yielded proofs of fifty-two of the sixty-three targets.

My journey now encountered, for a while, jungle too thick to penetrate. Indeed, various actions I took merely led to the discovery of the same fifty-two proofs. At the same time, while failures of the described sort were occurring, I (in my usual fashion) continued to conduct experiments resembling the first five already discussed. Three of those, the fourteenth and sixteenth and seventeenth, earlier-type experiments yielded some, though small, progress.

I then had a notion of how to cope with the encountered jungle. In particular, I adjoined to an input file, to be called 2a, quite like Input file 2, sixty-three resonators, each corresponding to one of the target theorems. To each of these resonators, I assigned the value 0, in contrast to an assignment of 1 for other resonators in the input file. The assignment of 0 caused the program to prefer for focus items that matched, with all variables treated as indistinguishable, one of the sixty-three targets. I also adjoined resonators corresponding to proof steps of proofs obtained in three experiments resembling the earlier-cited five. In addition, I assigned (in file 2a) the value 20 to max_weight, to permit OTTER to retain newly deduced formulas more complex than it had been retaining. Yes, the foliage parted some, and OTTER proved, with 2a, fifty-five of the desired sixty-three; the new proofs were for H35, H351, and H423. (By citing the specific theorems newly proved in an experiment, such as with file 2a, I provide you with material for testing and evaluating an approach you devise that is different from that under discussion; perhaps the later a theorem was proved, on the journey featured here, the harder it may be to prove.) For the curious, some of the theorems proved with Input File 2 and also proved in this small modified version (2a) of it required substantially more CPU time in the amended file, perhaps in part because of the assignment of 20 to max_weight.

I have referred at times to this research journey (in search of sixty-three proofs) as a labyrinth. My next experiment illustrates this labyrinthian nature and also illustrates (in effect) the type of guess I made as I traveled. As noted, I had in hand fifty-five of the sixty-three sought-after proofs. Among those I had not yet proved was HIF1. Why I focused on HIF1, rather than on some other theorem from among the eight still to be proved, I cannot say, other than a vague guess as to where to try to go next. The first move I made, in seeking a proof of HIF1, was to include in weight_list(pick_and_purge) 9 subformulas of the target, HIF1, omitting so-called trivial subformulas in the form of a variable. The next important move was to return to a breadth-first search with the command set(sos_queue), accompanied by assignments of appropriate (guessed) values to various parameters, 8 to max_weight, 200 to change_limit_after, and 6 to nex_max_weight. And so, among the important actions I took, I included three resonators that corresponded to a proof of HIF2, obtained with the first approach (in the eighteenth experiment that extended the spirit of the first five experiments). I in no way intend to suggest that a 3-step proof of HIF2 was in hand at this time, for the proof obtained in the cited eighteenth experiment relied very heavily on lemma adjunction in which many, many proof steps obtained from earlier runs were amended to the (initial) set of support. In a bit less than 1 CPU-hour, OTTER found the sought-after proof of HIF1, a proof of length 26 and level 6. Various other theorems among the sixty-three were also proved, but those were among the fifty-five whose proofs were already in hand.

I need to make totally clear that, in this second approach, the theorem HIF2 had not yet been proved, and yet I borrowed part of its proof (as resonators) from the first approach, which was still proceeding as I focused mainly on the second approach. I make no attempt to justify this move; rather, I suspect I am further illustrating the labyrinth I found myself in. You may, therefore, ask how you might proceed in partial or full emulation of what I am describing. Well, although perhaps not completely satisfying, you could proceed as I did, following two courses more or less simultaneously, each based on a different approach.

I now come to yet one more bit of evidence of the complexity present in this narrative. Indeed, in my next experiment, I returned to a search based in part on complexity (rather than solely on breadth-first), with an assignment of the value 2 to pick_given_ratio. Perhaps strange, I included 7 subformulas taken from the target named H3422. You naturally ask why H3422 came into play. Well, it was one of the now seven unproven (among the sixty-three) theorems. No doubt, you are anticipating, as I did (at this point), what would occur. You will be surprised, as I was, that H3422 was not proved; instead, HiF2 was the only new theorem that was proved, a proof of length 29 and level 13. That proof was completed in not much more than 36 CPU-seconds. Yes, searching for proofs is often far from straightforward.

To summarize where things were at this point on the journey, I note that six theorems remained to prove. They were H341, H3422, H432, H4441, HIF4, and HIF5. Therefore, theoretically, I had in hand fifty-seven proofs. However, I had not yet conducted an experiment that would, in one run, produce all fifty-seven. Indeed, the last two that were detailed each proved far fewer than fifty-seven.

I had conducted an experiment that proved in a single run both HIF1 and HIF2 and proved forty-six other theorems from among the sixty-three. So I conducted yet another experiment, rather like that using Input File 2, which I shall call 2b. I assigned 20 to max_weight with the goal of proving, perhaps, more than the fifty-seven so far proved. I included resonators for proofs of HIF1 and HIF2, as well as resonators from proofs obtained in runs based on the first approach and on the second. McCune's OTTER did present proofs of fifty-seven theorems, but the six cited as yet to prove remained unproved. Not much more than 5 CPU-minutes sufficed, suggesting that the resonators were providing the program with much power.

What, I wondered, should I try next, with the goal of proving one or more of the remaining six? After a few failures, I used an input file that was rather like that which yielded the fifty-seven proofs. However, I included resonators that corresponded to a proof of H3422, obtained with the twenty-second experiment that relied on the first approach. I included subformulas from H3422 and from HIF1. I did not include resonators that corresponded to a full proof of either HIF1 or HIF2. I also included resonators from proofs of many of the theorems already proved. In addition to, I hoped, finding proofs of one or more of the six left to prove, I assumed I would also be presented with proofs of all fifty-seven already in hand. Such was not the case; the run did prove fifty-eight of the sixty-three. In particular, of the fifty-seven, HIF1 was not proved. On the other hand, the experiment did prove H341 and H3422, which left four of the sixty-three to prove, namely, H432, H4441, HIF4, and HIF5.

Again, as a reminder, my pursuit and use of the so-called second approach was occurring as I continued to run experiments based on the first approach. One of those experiments, the twenty-third, yielded a proof of HIF4. I therefore, in the next experiment (based on the second approach), included resonators corresponding to a partial 13-step proof of HIF4. The cited proof was partial in that the twenty-third experiment (based on the first approach) relied heavily on lemma adjunction, with its list(sos) containing many proof steps from earlier proofs. In other words, I did not have a proof of HIF4 that relied solely on the ten Horn axioms. Nevertheless, the inclusion of resonators corresponding to a partial proof can, sometimes, be just what is needed to find a sought-after proof. In this case, evidence was supplied of the usefulness of partial proofs. OTTER produced a 72-step level-14 proof of HIF4, as well as proofs of almost all of the fifty-eight already proved. Again, a proof of HIF1 was not offered. But, at this point, only three of the sixty-three remained unproved. Also, I did not have an input file that completed proofs of all of the sixty theorems (out of sixty-three) already proved. Indeed, a proof of HIF1, although obtained earlier (as noted), was not offered by this last experiment.

As I so typically do, I addressed the easier problem first, namely, the production of an input file that, when used, would offer proofs of sixty theorems. I assigned max_weight the value 10, rather than 20, to enable the program to concentrate on less complex formulas, and I adjoined resonators corresponding to proof steps obtained in the experiment that completed fifty-nine proofs. Why adjoining those resonators would enable the program to prove HIF1, in addition to proving the others already proved, is still a mystery, especially in view of the fact that HIF1 had not been proved with the corresponding input file. Nevertheless, the modified input file, which I shall call 2c, succeeded, returning to me sixty (of the sixty-three) proofs being sought. (I generally do not pause to investigate such mysteries; perhaps the smaller max_weight played a key role.) I was therefore left to face the problem of finding three missing proofs.

My files indicate that, to break through in search of three missing proofs, I (in effect) combined results obtained with the second approach under discusssion, but used in a manner that the first approach followed. Specifically, as I was still engrossed in the second approach, I conducted an experiment relying on a breadth-first search and a (full) lemma-adjunction methodology; indeed, I adjoined 271 proof steps obtained, with 2c, from proofs found with the input file that yielded sixty proofs. (You see again how labyrinthian this research journey was.) This experiment was the twenty-fifth conducted with approach 1. The experiment yielded proofs of H4441 and HIF5, more accurately, partial proofs in that many lemmas were employed in the inpt file. Nevertheless, in emulation of what I discussed earlier, I then used the two partial proofs in yet another (second approach) experiment. In particular, I adjoined thirteen resonators, sorted proof steps from the two partial proofs, and I assigned the value 6 to max_weight and proceeded. Again, I cannot explain precisely why I chose such a small value for max_weight. The experiment under discussion, that relied on the cited two partial proofs, proved most satisfying; both H4441 and HIF5 were proved, along with the sixty that had been proved just a bit earlier on the trip. In other words, I had an input file, to be called 2d, that presented to me proofs of sixty-two of the sixty-three theorems that motivated the long, long journey.

To obtain the last of the sixty-three proofs, an obvious move sufficed. Indeed, I had commented out the resonators corresponding to the sixty-three targets. By now commenting back in the items, in other words by including as a resonator the formula H432, and submitting the slightly amended input file, OTTER was able to supply me with the desired sixty-three proofs. Indeed, the formula, H342, had been deduced, but it was not retained because of the small value, 6, assigned to max_weight. When the corresponding resonator was included, with an assigned value much smaller than 6, the formula when deduced was in fact retained, which enabled OTTER to complete the sought-after sixty-third proof.

You now have a general methodology for seeking proofs of a fairly large collection of theorems, a collection that might be thought of as tightly connected. Of course, these sixty-three theorems had been proved decades ago. The fact that their proofs could be found by hand does not in any way detract from the fact that an automated reasoning program, namely, McCune's OTTER, was able to find all of the desired proofs. Indeed, as noted early in this notebook, attempting to meet the challenge of finding proofs with such a program, when some or many resist the effort, merits time and energy. The successful dispatching of such challenges can, and often does, lead to sharply improved power of reasoning programs. With that greater power, sometimes, questions that had been open for years are answered, proofs showing that certain thought-to-be indispensable lemmas or axioms are discovered without the use of said lemmas or axioms, and more elegant proofs are completed.

You might naturally wonder whether the attempt to prove theorems whose proof is already known has just theoretical value, in contrast to actual value. The following story will, for some individuals, answer the implied question in the affirmative. On the other hand, perhaps this small narrative will simply produce skepticism. The area of logic of concern is axiomatized by three formulas, *B*, *C*, and *I*, the following.

```
P(i(i(u,v),i(i(w,u),i(w,v)))). % B
```

```
P(i(i(u,i(v,w)),i(v,i(u,w)))). % C
P(i(u,u)). % I
```

John Halleck had found, with his program shotgun, that the following formula is a single axiom for this logic; his was a new discovery.

```
P(i(i(i(x,x),i(y,i(z,u))),i(i(u,v),i(z,i(y,v))))).
```

I wondered (I believe in March 2009) whether the use of the rather recently formulated subformla strategy would enable OTTER to independently find a proof establishing the formula to be a single axiom. I, therefore, put together an input file that relied on the use of subformulas from the given—and eventually proved—single axiom as well as subformulas from each of *B*, *C*, and *I*. No other resonators were included. In a bit less than 1700 CPU-seconds, OTTER produced the desired proof. My conclusion, perhaps not shared by all, was that the use of the subformula strategy would have found a new single axiom, without knowledge of Halleck's fine research. Further, I concluded that the attempt to find, with the aid of a reasoning program, proofs of theorems that had already been proved was one of the various fine ways of seeking more power for such programs. After all, the study of a Rezus-style single axiom led to the formulation of the subformula strategy.

Beeson's challenge had been met; indeed, the sixty-three target theorems had each been proved, proved, as required, with an automated reasoning program. The following input file suffices.

Input File 3

```
set(hyper_res).
assign(max_weight,6).
% assign(change_limit_after,200).
% assign(new_max_weight,6).
clear(print_kept).
% set(process_input).
% set(ancestor_subsume).
set(back_sub).
clear(print_back_sub).
assign(max_distinct_vars,6).
assign(pick_given_ratio,2).
assign(max_mem,1900000).
% assign(max_seconds,1).
% assign(max_given,24).
assign(max_proofs,-1).
assign(report,5400).
set(order_history).
set(input_sos_first).
% set(sos_queue).
set(keep_hint_subsumers).
assign(bsub_hint_wt,1).

weight_list(pick_and_purge).
% Following 1 proves H432, out9y5
weight(P(i(n(n(n(x))),n(x))),0).
% Following sorted 13 prove HIF5 and 4441 .out9y.
weight(P(a(x,i(n(a(y,z)),k(n(y),n(z))))),0).
weight(P(i(a(x,x),k(x,i(k(n(y),n(z)),n(a(y,z)))))),0).
weight(P(i(i(a(x,x),y),i(a(x,x),k(x,y)))),0).
weight(P(i(i(k(x,y),z),i(i(x,y),i(x,z)))),0).
weight(P(i(i(n(a(x,y)),n(z)),i(n(a(x,y)),k(n(x),n(z))))),0).
```

```
weight(P(i(i(x,y),i(k(n(y),n(z)),k(n(x),n(z))))),0).
weight(P(i(i(x,y),i(k(n(y),n(z)),n(x)))),0).
weight(P(i(k(n(a(x,y)),n(z)),k(n(x),n(z)))),0).
weight(P(i(n(a(x,y)),k(n(x),n(y)))),0).
weight(P(i(n(a(x,y)),n(y))),0).
weight(P(i(n(x),i(i(y,x),i(y,z)))),0).
weight(P(i(x,i(k(n(y),n(z)),n(a(y,z))))),0).
weight(P(k(i(n(a(x,y)),k(n(x),n(y))),i(k(n(z),n(u)),n(a(z,u))))),0).
% Following 13 prove HIF1 and HIF4, .out9w.
weight(P(i(i(i(k(x,y),x),i(i(k(z,u),u),v)),v)),0).
weight(P(i(i(i(x,y),z),i(i(y,u),i(i(x,y),k(z,i(x,u)))))),0).
weight(P(i(i(x,k(y,z)),i(x,k(y,x)))),0).
weight(P(i(i(x,k(y,z)),k(i(x,y),i(x,z)))),0).
weight(P(i(i(x,y),i(i(x,z),i(i(u,x),k(i(u,y),i(u,z)))))),0).
weight(P(i(i(x,y),k(i(k(i(z,u),i(z,v)),i(z,k(u,v))),i(i(x,w),i(x,k(y,w)))))),0).
weight(P(i(i(x,y),k(i(k(i(z,u),i(z,v)),i(z,k(u,v))),i(x,y)))),0).
weight(P(i(k(i(x,y),i(x,z)),i(x,k(y,z)))),0).
weight(P(i(k(x,y),k(y,i(k(a(z,u),a(v,u)),a(k(z,v),u))))),0).
weight(P(i(x,i(i(y,z),k(x,i(i(y,u),i(y,k(z,u))))))),0).
weight(P(k(i(a(k(x,y),z),k(a(x,z),a(y,z))),i(a(k(x,y),z),k(a(x,z),a(y,z))))),0).
weight(P(k(i(a(k(x,y),z),k(a(x,z),a(y,z))),i(k(a(u,v),a(w,v)),a(k(u,w),v)))),0).
weight(P(k(i(k(i(x,y),i(x,z)),i(x,k(y,z))),i(i(u,k(v,w)),k(i(u,v),i(u,w))))),0).
% Following 223 sorted .out99k proof steps from .out99j.
weight(P(i(a(a(x,y),z),a(a(y,x),z))),1).
weight(P(i(a(a(x,y),z),a(a(y,z),x))),1).
weight(P(i(a(a(x,y),z),a(x,a(y,z)))),1).
weight(P(i(a(k(x,n(x)),y),y)),1).
weight(P(i(a(k(x,y),k(z,y)),k(a(x,z),y))),1).
weight(P(i(a(k(x,y),z),a(x,z))),1).
weight(P(i(a(k(x,y),z),a(y,z))),1).
weight(P(i(a(k(x,y),z),k(a(x,z),a(y,z)))),1).
weight(P(i(a(n(x),y),i(x,y))),1).
weight(P(i(a(x,a(y,z)),a(a(x,y),z))),1).
weight(P(i(a(x,a(y,z)),a(a(z,x),y))),1).
weight(P(i(a(x,x),x)),1).
weight(P(i(a(x,y),a(a(y,x),z))),1).
weight(P(i(a(x,y),a(a(y,z),x))),1).
weight(P(i(a(x,y),a(a(z,x),y))),1).
weight(P(i(a(x,y),a(a(z,y),x))),1).
weight(P(i(a(x,y),a(y,x))),1).
weight(P(i(a(x,y),i(i(x,y),y))),1).
weight(P(i(a(x,y),i(n(x),y))),1).
weight(P(i(a(x,y),i(n(y),x))),1).
weight(P(i(i(a(k(x,y),z),u),i(a(k(x,y),z),k(a(x,z),u)))),1).
weight(P(i(i(a(x,y),z),i(a(y,x),z))),1).
weight(P(i(i(a(x,y),z),i(x,z))),1).
weight(P(i(i(i(i(x,a(y,z)),i(a(u,x),a(y,z))),v),i(i(u,y),v))),1).
weight(P(i(i(i(i(x,y),i(x,k(z,y))),u),i(i(x,z),u))),1).
weight(P(i(i(i(i(x,y),i(x,z)),u),i(i(y,z),u))),1).
weight(P(i(i(i(i(x,y),i(z,y)),u),i(i(z,x),u))),1).
weight(P(i(i(i(k(x,y),z),u),i(i(x,i(y,z)),u))),1).
weight(P(i(i(i(k(x,y),z),u),i(i(x,z),u))),1).
weight(P(i(i(i(k(x,y),z),u),i(i(y,i(x,z)),u))),1).
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weight(P(i(i(i(k(x,y),z),u),i(i(y,z),u))),1).
weight(P(i(i(i(x,a(x,y)),z),z)),1).
weight(P(i(i(i(x,a(y,x)),z),z)),1).
weight(P(i(i(i(x,a(y,z)),u),i(i(x,y),u))),1).
weight(P(i(i(i(x,a(y,z)),u),i(i(x,z),u))),1).
weight(P(i(i(i(x,i(y,z)),i(x,y)),i(i(x,i(y,z)),i(x,z)))),1).
weight(P(i(i(i(x,i(y,z)),u),i(i(y,i(x,z)),u))),1).
weight(P(i(i(i(x,k(x,y)),z),i(y,z))),1).
weight(P(i(i(i(x,k(y,x)),z),i(y,z))),1).
weight(P(i(i(i(x,k(y,z)),u),i(k(y,i(x,z)),u))),1).
weight(P(i(i(i(x,y),i(x,z)),i(i(x,y),i(x,k(y,z))))),1).
weight(P(i(i(i(x,y),i(z,y)),i(i(x,y),i(a(x,z),y)))),1).
weight(P(i(i(k(x,i(y,z)),y),i(k(x,i(y,z)),z))),1).
weight(P(i(i(k(x,k(y,z)),u),i(k(k(x,y),z),u))),1).
weight(P(i(i(k(x,k(y,z)),u),i(k(x,k(y,z)),k(k(x,y),u)))),1).
weight(P(i(i(k(x,n(y)),y),i(k(x,n(y)),z))),1).
weight(P(i(i(k(x,y),a(z,u)),i(k(a(y,u),x),a(z,u)))),1).
weight(P(i(i(k(x,y),a(z,u)),i(k(x,y),a(k(u,y),z)))),1).
weight(P(i(i(k(x,y),a(z,u)),i(k(x,y),a(k(y,z),u)))),1).
weight(P(i(i(k(x,y),a(z,u)),i(k(x,y),a(k(z,y),u)))),1).
weight(P(i(i(k(x,y),i(x,z)),i(k(x,y),z))),1).
weight(P(i(i(k(x,y),i(y,z)),i(i(k(x,y),u),i(k(x,y),k(z,u))))),1).
weight(P(i(i(k(x,y),i(y,z)),i(k(x,y),z))),1).
weight(P(i(i(k(x,y),i(z,u)),i(k(k(z,x),y),u))),1).
weight(P(i(i(k(x,y),z),i(i(u,k(z,y)),i(a(k(x,y),u),k(z,y))))),1).
weight(P(i(i(k(x,y),z),i(i(y,u),i(k(x,y),k(z,u))))),1).
weight(P(i(i(k(x,y),z),i(k(x,n(z)),n(y)))),1).
weight(P(i(i(k(x,y),z),i(k(x,y),k(x,z)))),1).
weight(P(i(i(k(x,y),z),i(k(x,y),k(y,z)))),1).
weight(P(i(i(k(x,y),z),i(k(x,y),k(z,x)))),1).
weight(P(i(i(k(x,y),z),i(k(x,y),k(z,y)))),1).
weight(P(i(i(k(x,y),z),i(x,i(y,z)))),1).
weight(P(i(i(x,a(y,z)),i(a(x,a(u,y)),a(a(y,z),u)))),1).
weight(P(i(i(x,a(y,z)),i(a(x,a(z,u)),a(a(y,z),u)))),1).
weight(P(i(i(x,a(y,z)),i(a(x,y),a(y,z)))),1).
weight(P(i(i(x,a(y,z)),i(a(x,z),a(y,z)))),1).
weight(P(i(i(x,a(y,z)),i(a(z,x),a(y,z)))),1).
weight(P(i(i(x,a(y,z)),i(i(u,y),i(a(u,x),a(y,z))))),1).
weight(P(i(i(x,a(y,z)),i(i(y,z),i(x,z)))),1).
weight(P(i(i(x,a(y,z)),i(k(i(y,u),i(z,u)),i(x,u)))),1).
weight(P(i(i(x,a(y,z)),i(k(x,u),a(z,y)))),1).
weight(P(i(i(x,a(y,z)),i(x,a(z,y)))),1).
weight(P(i(i(x,i(i(k(y,z),y),u)),i(x,u))),1).
weight(P(i(i(x,i(i(k(y,z),z),u)),i(x,u))),1).
weight(P(i(i(x,i(i(y,a(y,z)),u)),i(x,u))),1).
weight(P(i(i(x,i(i(y,a(z,y)),u)),i(x,u))),1).
weight(P(i(i(x,i(i(y,i(z,y)),u)),i(x,u))),1).
weight(P(i(i(x,i(i(y,x),z)),i(x,z))),1).
weight(P(i(i(x,i(k(y,i(z,u)),z)),i(x,i(k(y,i(z,u)),u)))),1).
weight(P(i(i(x,i(k(y,z),u)),i(k(k(x,y),z),u))),1).
weight(P(i(i(x,i(k(y,z),u)),i(x,i(k(y,z),k(u,y))))),1).
weight(P(i(i(x,i(k(y,z),u)),i(x,i(k(y,z),k(u,z))))),1).
weight(P(i(i(x,i(k(y,z),u)),i(x,i(k(y,z),k(y,u))))),1).

weight(P(i(i(x,i(y,i(z,i(u,v)))),i(x,i(y,i(k(z,u),v))))),1).
weight(P(i(i(x,i(y,i(z,n(x)))),i(x,i(y,n(z))))),1).
weight(P(i(i(x,i(y,i(z,u))),i(x,i(i(y,z),i(y,u))))),1).
weight(P(i(i(x,i(y,i(z,u))),i(y,i(k(x,n(u)),n(z))))),1).
weight(P(i(i(x,i(y,i(z,u))),i(y,i(x,i(n(u),n(z)))))),1).
weight(P(i(i(x,i(y,n(x))),i(x,n(y)))),1).
weight(P(i(i(x,i(y,n(y))),i(x,n(y)))),1).
weight(P(i(i(x,i(y,z)),i(a(n(y),x),i(y,z)))),1).
weight(P(i(i(x,i(y,z)),i(a(x,z),i(y,z)))),1).
weight(P(i(i(x,i(y,z)),i(a(z,x),i(y,z)))),1).
weight(P(i(i(x,i(y,z)),i(i(k(u,x),y),i(k(u,x),z)))),1).
weight(P(i(i(x,i(y,z)),i(i(k(x,u),y),i(k(x,u),z)))),1).
weight(P(i(i(x,i(y,z)),i(i(u,i(x,y)),i(u,i(x,z))))),1).
weight(P(i(i(x,i(y,z)),i(i(u,x),i(y,i(u,z))))),1).
weight(P(i(i(x,i(y,z)),i(i(u,y),i(x,i(u,z))))),1).
weight(P(i(i(x,i(y,z)),i(i(x,i(u,y)),i(x,i(u,z))))),1).
weight(P(i(i(x,i(y,z)),i(i(y,i(u,x)),i(y,i(u,z))))),1).
weight(P(i(i(x,i(y,z)),i(i(y,x),i(y,z)))),1).
weight(P(i(i(x,i(y,z)),i(i(z,u),i(x,i(y,u))))),1).
weight(P(i(i(x,i(y,z)),i(k(x,y),z))),1).
weight(P(i(i(x,i(y,z)),i(k(y,x),z))),1).
weight(P(i(i(x,i(y,z)),i(u,i(i(x,y),i(x,z))))),1).
weight(P(i(i(x,i(y,z)),i(x,i(i(u,y),i(u,z))))),1).
weight(P(i(i(x,i(y,z)),i(x,i(i(u,z),i(a(y,u),z))))),1).
weight(P(i(i(x,i(y,z)),i(x,i(i(y,u),i(y,k(z,u)))))),1).
weight(P(i(i(x,i(y,z)),i(x,i(i(z,u),i(y,u))))),1).
weight(P(i(i(x,i(y,z)),i(x,i(n(z),n(y))))),1).
weight(P(i(i(x,i(y,z)),i(y,i(x,z)))),1).
weight(P(i(i(x,k(a(y,z),u)),i(a(k(y,u),x),k(a(y,z),u)))),1).
weight(P(i(i(x,k(i(y,z),n(z))),i(x,n(y)))),1).
weight(P(i(i(x,n(x)),i(x,y))),1).
weight(P(i(i(x,n(x)),n(x))),1).
weight(P(i(i(x,y),i(a(x,y),y))),1).
weight(P(i(i(x,y),i(a(x,z),a(y,z)))),1).
weight(P(i(i(x,y),i(a(y,x),y))),1).
weight(P(i(i(x,y),i(a(z,x),a(y,z)))),1).
weight(P(i(i(x,y),i(i(y,z),i(x,z)))),1).
weight(P(i(i(x,y),i(i(z,a(u,y)),i(a(x,z),a(u,y))))),1).
weight(P(i(i(x,y),i(i(z,a(y,u)),i(a(x,z),a(y,u))))),1).
weight(P(i(i(x,y),i(i(z,i(u,x)),i(u,i(z,y))))),1).
weight(P(i(i(x,y),i(i(z,i(u,x)),i(z,i(u,y))))),1).
weight(P(i(i(x,y),i(i(z,u),i(a(x,z),a(y,u))))),1).
weight(P(i(i(x,y),i(i(z,u),i(k(x,z),k(y,u))))),1).
weight(P(i(i(x,y),i(i(z,x),i(z,a(y,u))))),1).
weight(P(i(i(x,y),i(i(z,x),i(z,y)))),1).
weight(P(i(i(x,y),i(k(x,z),a(y,u)))),1).
weight(P(i(i(x,y),i(k(x,z),k(y,z)))),1).
weight(P(i(i(x,y),i(k(x,z),y))),1).
weight(P(i(i(x,y),i(k(z,x),y))),1).
weight(P(i(i(x,y),i(n(n(x)),n(n(y))))),1).
weight(P(i(i(x,y),i(n(y),i(x,z)))),1).
weight(P(i(i(x,y),i(n(y),n(x)))),1).
weight(P(i(i(x,y),i(x,a(y,z)))),1).

weight(P(i(i(x,y),i(x,a(z,y)))),1).
weight(P(i(i(x,y),i(x,i(z,y)))),1).
weight(P(i(i(x,y),i(x,k(x,y)))),1).
weight(P(i(i(x,y),i(x,x))),1).
weight(P(i(i(x,y),i(z,i(i(x,u),i(x,k(y,u)))))),1).
weight(P(i(i(x,y),i(z,i(x,a(y,u))))),1).
weight(P(i(i(x,y),i(z,i(x,x)))),1).
weight(P(i(k(a(x,y),a(z,y)),a(k(x,z),y))),1).
weight(P(i(k(a(x,y),n(x)),y)),1).
weight(P(i(k(a(x,y),z),a(k(x,z),k(y,z)))),1).
weight(P(i(k(a(x,y),z),a(k(x,z),y))),1).
weight(P(i(k(a(x,y),z),a(k(y,z),k(x,z)))),1).
weight(P(i(k(a(x,y),z),a(k(z,x),a(u,y)))),1).
weight(P(i(k(a(x,y),z),a(k(z,x),y))),1).
weight(P(i(k(a(x,y),z),a(x,a(u,y)))),1).
weight(P(i(k(i(x,y),i(x,n(y))),n(x))),1).
weight(P(i(k(i(x,y),i(y,z)),i(x,z))),1).
weight(P(i(k(i(x,y),i(z,n(y))),i(z,n(x)))),1).
weight(P(i(k(i(x,y),i(z,u)),i(a(x,z),a(y,u)))),1).
weight(P(i(k(i(x,y),i(z,u)),i(i(v,a(x,z)),i(v,a(y,u))))),1).
weight(P(i(k(i(x,y),i(z,u)),i(k(x,z),k(y,u)))),1).
weight(P(i(k(i(x,y),i(z,y)),i(a(x,z),y))),1).
weight(P(i(k(i(x,y),n(y)),n(x))),1).
weight(P(i(k(i(x,y),z),i(i(u,y),i(a(x,u),y)))),1).
weight(P(i(k(k(i(x,a(y,z)),i(y,u)),i(z,u)),i(x,u))),1).
weight(P(i(k(k(i(x,a(y,z)),i(y,u)),i(z,v)),i(x,a(u,v)))),1).
weight(P(i(k(k(x,k(y,z)),i(z,u)),i(v,u))),1).
weight(P(i(k(k(x,k(y,z)),i(z,u)),u)),1).
weight(P(i(k(k(x,k(y,z)),u),z)),1).
weight(P(i(k(k(x,y),z),k(k(y,x),z))),1).
weight(P(i(k(k(x,y),z),k(x,k(y,z)))),1).
weight(P(i(k(n(x),n(y)),n(a(x,y)))),1).
weight(P(i(k(x,i(k(x,y),z)),i(y,z))),1).
weight(P(i(k(x,i(x,y)),y)),1).
weight(P(i(k(x,i(y,z)),i(i(u,y),i(u,z)))),1).
weight(P(i(k(x,i(y,z)),i(y,k(x,z)))),1).
weight(P(i(k(x,k(y,z)),k(k(x,y),z))),1).
weight(P(i(k(x,k(y,z)),z)),1).
weight(P(i(k(x,n(x)),y)),1).
weight(P(i(k(x,y),a(x,z))),1).
weight(P(i(k(x,y),a(z,x))),1).
weight(P(i(k(x,y),i(i(z,u),i(z,k(x,u))))),1).
weight(P(i(k(x,y),i(k(z,u),k(x,z)))),1).
weight(P(i(k(x,y),i(z,x))),1).
weight(P(i(k(x,y),i(z,y))),1).
weight(P(i(k(x,y),k(a(z,x),y))),1).
weight(P(i(k(x,y),k(y,x))),1).
weight(P(i(n(x),i(a(x,y),y))),1).
weight(P(i(n(x),i(n(y),n(a(x,y))))),1).
weight(P(i(x,a(a(x,y),z))),1).
weight(P(i(x,a(a(y,x),z))),1).
weight(P(i(x,i(a(y,z),a(k(x,y),z)))),1).
weight(P(i(x,i(a(y,z),a(k(y,x),z)))),1).

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weight(P(i(x,i(a(y,z),a(k(z,x),y)))),1).
weight(P(i(x,i(i(i(y,z),i(u,y)),i(i(y,z),i(u,z))))),1).
weight(P(i(x,i(i(k(x,y),z),i(y,z)))),1).
weight(P(i(x,i(i(k(y,z),u),i(k(y,z),k(y,u))))),1).
weight(P(i(x,i(i(k(y,z),u),i(k(y,z),k(z,u))))),1).
weight(P(i(x,i(i(y,i(z,u)),i(i(y,z),i(y,u))))),1).
weight(P(i(x,i(i(y,z),i(i(u,z),i(a(y,u),z))))),1).
weight(P(i(x,i(i(y,z),i(i(y,u),i(y,k(z,u)))))),1).
weight(P(i(x,i(i(y,z),i(y,k(y,z))))),1).
weight(P(i(x,i(k(y,i(k(y,x),z)),z))),1).
weight(P(i(x,i(k(y,z),k(y,k(z,x))))),1).
weight(P(i(x,i(k(y,z),k(y,x)))),1).
weight(P(i(x,i(k(y,z),k(z,x)))),1).
weight(P(i(x,i(k(y,z),y))),1).
weight(P(i(x,i(k(y,z),z))),1).
weight(P(i(x,i(n(x),y))),1).
weight(P(i(x,i(y,a(y,z)))),1).
weight(P(i(x,i(y,a(z,y)))),1).
weight(P(i(x,i(y,i(k(z,u),u)))),1).
weight(P(i(x,i(y,i(k(z,u),z)))),1).
weight(P(i(x,i(y,i(z,a(u,z))))),1).
weight(P(i(x,i(y,i(z,a(z,u))))),1).
weight(P(i(x,i(y,i(z,i(u,z))))),1).
weight(P(i(x,i(y,i(z,y)))),1).
weight(P(i(x,i(y,k(x,y)))),1).
weight(P(i(x,i(y,k(y,x)))),1).
weight(P(i(x,i(y,y))),1).
weight(P(i(x,k(x,i(i(k(y,z),u),i(y,i(z,u)))))),1).
weight(P(i(x,k(x,i(i(y,i(z,u)),i(z,i(y,u)))))),1).
weight(P(i(x,k(x,x))),1).
weight(P(i(x,n(n(x)))),1).
weight(P(i(x,x)),1).
weight(P(k(i(i(x,i(y,z)),i(k(x,y),z)),i(i(k(u,v),w),i(u,i(v,w))))),1).
weight(P(k(i(i(x,i(y,z)),i(y,i(x,z))),i(i(u,i(v,w)),i(v,i(u,w))))),1).
weight(junk,1000).
end_of_list.

list(usable).
% condensed detachment
-P(i(x,y)) | -P(x) | P(y).
%  P(i(k(x,y),y)).      % 4
%  P(i(i(x,y),i(i(x,z),i(x,k(y,z))))).   % 5
P(i(x,a(x,y))).           % 6
P(i(x,a(y,x))).           % 7
P(i(i(x,y),i(i(z,y),i(a(x,z),y)))). % 8
%  P(i(i(x,n(y)),i(y,n(x)))).    % 9
%  P(i(n(x),i(x,y))).      % 10
end_of_list.

list(sos).
%  Following 10 is an axiomatization of intuitionistic logic from Horn's 1962 paper
P(i(x,i(y,x))).      % 1
P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).   % 2
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P(i(k(x,y),x)).     % 3
P(i(k(x,y),y)).     % 4
P(i(i(x,y),i(i(x,z),i(x,k(y,z))))).   % 5
%  P(i(x,a(x,y))).          % 6
%  P(i(x,a(y,x))).          % 7
%  P(i(i(x,y),i(i(z,y),i(a(x,z),y)))). % 8
P(i(i(x,n(y)),i(y,n(x)))).   % 9
P(i(n(x),i(x,y))).      % 10
end_of_list.

list(passive).
%  Following are eight of Heyting's axioms and four others.
-P(i(b,k(b,b))) | $ANS(01).
-P(i(k(b,c),k(c,b))) | $ANS(02).
-P(i(i(b,c),i(k(b,d),k(c,d)))) | $ANS(03).
-P(i(k(i(b,c),i(c,d)),i(b,d))) | $ANS(04).
-P(i(k(b,i(b,c)),c)) | $ANS(06).
-P(i(a(b,c),a(c,b))) | $ANS(08).
-P(i(k(i(b,d),i(c,d)),i(a(b,c),d))) | $ANS(09).
-P(i(k(i(b,c),i(b,n(c))),n(b))) | $ANS(11).
-P(i(i(q,r),i(i(p,q),i(p,r)))) | $ANS(H2).
-P(i(i(p,i(q,r)),i(q,i(p,r)))) | $ANS(H3).
-P(i(i(p,n(p)),n(p)))          | $ANS(N1).
-P(i(p,i(n(p),q))) | $ANS(N2).
%  Following theorems from Heyting's paper.
-P(i(p,p)) | $ANS(H221).
-P(i(k(i(p,q),i(r,d)),i(k(p,r),k(q,d)))) | $ANS(H223).
%  -P(iff(k(i(p,q),i(p,r)),i(p,k(q,r)))) | $ANS(H224).
-P(i(k(p,i(q,r)),i(q,k(p,r)))) | $ANS(H225).
-P(i(p,i(q,k(q,p)))) | $ANS(H226).
%  -P(iff(i(p,i(b,c)),i(k(p,b),c))) | $ANS(H227).
%  -P(iff(i(p,i(b,c)),i(b,i(p,c)))) | $ANS(H2271).
-P(i(i(p,c),i(k(p,b),c))) | $ANS(H228).
-P(i(i(p,b),i(p,i(c,b)))) | $ANS(H2281).
-P(i(k(p,i(k(p,b),c)),i(b,c))) | $ANS(H2282).
-P(i(i(p,b),i(i(b,c),i(p,c)))) | $ANS(H229).
-P(i(i(b,c),i(i(p,b),i(p,c)))) | $ANS(H2291).
-P(i(k(k(p,b),c),k(p,k(b,c)))) | $ANS(H23).
-P(i(k(k(p,b),c),k(k(b,p),c))) | $ANS(H231).
-P(i(k(p,k(b,c)),k(k(p,b),c))) | $ANS(H232).
-P(i(k(k(i(p,b),k(b,c)),i(c,d)),i(p,d))) | $ANS(H24).
-P(i(a(a(p,b),c),a(p,a(b,c)))) | $ANS(H32).
-P(i(a(p,a(b,c)),a(a(p,b),c))) | $ANS(H321).
-P(i(a(p,p),p))            | $ANS(H322).
-P(i(k(i(p,b),i(c,d)),i(a(p,c),a(b,d)))) | $ANS(H33).
-P(i(i(p,b),i(k(p,c),a(b,d)))) | $ANS(H331).
-P(i(i(p,b),i(a(p,b),b)))    | $ANS(H332).
-P(i(i(a(p,b),b),i(p,b)))    | $ANS(H333).
-P(i(i(p,b),i(a(p,c),a(b,c)))) | $ANS(H334).
-P(i(a(a(p,b),c),a(a(b,p),c))) | $ANS(H335).
-P(i(a(a(b,p),c),a(a(p,c),b))) | $ANS(H3351).
-P(i(a(k(p,c),k(b,c)),k(a(p,b),c))) | $ANS(H34).
-P(i(k(a(p,b),c),a(k(p,c),k(b,c)))) | $ANS(H341).
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-P(i(a(k(p,b),c),k(a(p,c),a(b,c)))) | $ANS(H3421).
-P(i(k(a(p,c),a(b,c)),a(k(p,b),c))) | $ANS(H3422).
%  -P(iff(a(k(p,b),c),k(a(p,c),a(b,c)))) | $ANS(H342).
-P(i(k(k(i(p,a(b,c)),i(b,d)),i(c,e)),i(p,a(d,e)))) | $ANS(H35).
-P(i(k(k(i(p,a(b,c)),i(b,d)),i(c,d)),i(p,d))) | $ANS(H351).
-P(i(a(p,b),i(i(p,b),b)))  | $ANS(H36).
-P(i(i(p,b),i(n(b),n(p)))) | $ANS(H42).
-P(i(i(p,b),i(n(n(p)),n(n(b))))) | $ANS(H422).
-P(i(i(k(p,b),c),i(k(p,n(c)),n(b)))) | $ANS(H423).
-P(i(k(i(p,b),n(b)),n(p))) | $ANS(H424).
-P(i(p,n(n(p)))) | $ANS(H43).
-P(i(n(p),n(n(n(p))))) | $ANS(H431).
-P(i(n(n(n(p))),n(p))) | $ANS(H432).
-P(i(k(p,n(p)),b)) | $ANS(H44).
-P(i(a(k(p,n(p)),b),b)) | $ANS(H441).
-P(i(k(a(p,b),n(p)),b)) | $ANS(H442).
-P(i(n(p),i(n(b),n(a(p,b))))) | $ANS(H443).
-P(i(n(a(p,b)),k(n(p),n(b)))) | $ANS(H4441).
-P(i(k(n(p),n(b)),n(a(p,b)))) | $ANS(H4442).
%  -P(iff(n(a(p,b)),k(n(p),n(b)))) | $ANS(H444).
-P(i(a(p,n(p)),i(n(n(p)),p))) | $ANS(H445).
-P(i(a(n(p),b),i(p,b))) | $ANS(H446).
-P(i(a(p,b),i(n(p),b))) | $ANS(H447).
%  Following 5 are expanded theorems corresponding to iff theorems.
-P(k(i(k(i(p,q),i(p,r)),i(p,k(q,r))),i(i(p,k(q,r)),k(i(p,q),i(p,r))))) | $ANS(HIF1).
-P(k(i(i(p,i(q,r)),i(k(p,q),r)),i(i(k(p,q),r),i(p,i(q,r))))) | $ANS(HIF2).
-P(k(i(i(p,i(q,r)),i(q,i(p,r))),i(i(q,i(p,r)),i(p,i(q,r))))) | $ANS(HIF3).
-P(k(i(a(k(p,q),r),k(a(p,r),a(q,r))),i(k(a(p,r),a(q,r)),a(k(p,q),r)))) | $ANS(HIF4).
-P(k(i(n(a(p,q)),k(n(p),n(q))),i(k(n(p),n(q)),n(a(p,q))))) | $ANS(HIF5).
end_of_list.

list(demodulators).
%  n(n(n(x))) = junk.
%  iff(x,y) = k(i(x,y),i(y,x)).
%  n(n(x)) = junk.
i(x,junk) = junk.
i(junk,x) = junk.
n(junk) = junk.
end_of_list.
```

Further, the additional challenge of producing a methodology that might be of use in contexts that included automated finding of proofs for some loosely connected set of theorems was now in hand. (At this point, the original journey has ended; but you might enjoy embarking, as I did, on the related journey that is now chronicled.) Perhaps the sane thing to do was to turn to a sharply different bit of research. But, as some of you might guess, I was almost immediately ready to embark on a new journey, the objective of which was to find “short” proofs for each of the target theorems. Little did I know, as I boarded the ship, how my goals would shift, what obstacles I would encounter, and where the journey would take me. The details of my sojourn are next presented to you.

5. Searching for Elegant Proofs

Although you might have little or no interest in proofs that are shorter than found in the literature, that avoid some thought-to-be indispensable lemma, or that are more elegant in some manner, this section could, nevertheless, provide you with most useful approaches. Indeed, some of the actions I took on the journey that I now report can be most profitably used in finding first proofs and in other contexts. After all, the search space that often must be examined is gigantic (whether seeking a first proof or seeking a shorter proof), and, therefore, procedures and actions that permit a reasoning program to effectively cope with this obstacle merit study. By the way, if you search the literature, you will find that Hilbert himself was interested, very interested, in finding simpler proofs; that topic is known, in some papers, as Hilbert's 24th problem. Indeed, as evidence of Hilbert's concern for finding simpler proofs, in 1917 Hilbert cited the problem in one of his lectures.

I have spent much of my time in the past decade searching for elegant proofs, mainly, but not exclusively, in the context of length. McCune—yes, the author of OTTER—because of my consuming interest in proof length provided me with a marvelous bit of computer code. To invoke the code, you include the command set(ancestor_subsume). You saw that command in the three given input files, but commented out (with a percent sign). Ancestor subsumption offers you an automated means for seeking shorter proofs; it is a procedure that compares derivation lengths to the same conclusion and prefers the strictly shorter. This procedure is by no means a panacea; but its use, when the journey focuses on finding shorter and ever shorter proofs, provides a splendid jumping off point.

For so many, many years, I have begun the search for shorter proofs by relying heavily on McCune's ancestor subsumption. Therefore, in the beginning of my journey that was to last more than five months, I took Input File 3 and made but two changes. I assigned the value 10 to max_weight, rather than 6, and I invoked McCune's powerful procedure by merely removing the percent sign. Yes, I expected to see progress in the context of proof shortening with this amended file, 3a. And indeed I did. With Input file 3, OTTER found an 84-step proof of HIF4. With 3a, the program found a 68-step proof of this theorem. In place of a 77-step proof of HIF5, the use of 3a led to a 70-step proof. With 3, the program found a 59-step proof of both H3422 and H4441; with 3a, OTTER found, respectively, proofs of length 52 and 53. Of the sixty-three proofs, forty-five of the new proofs were of a different length with 3a than with 3, forty-one shorter and, surprise, four longer. Among the more satisfying proof reductions were those for H432 and H44, each reduced from a length of 23 to a length of 7. But, I can guess, you wonder how the use of ancestor subsumption can lead to finding a longer proof. A word of warning: One can construct examples in which shorter subproofs do not necessarily make a shorter total proof; for examples of how this can occur, see the notebook entitled **The Joy of Solving a Decades-Old Mystery: Success with the BCK and BCI Logics.***

Just after noon on February 13, 2010, as the results from 3a (found for me in .out99q) were obtained, I turned to a file that I shall call 3b (for me, in99r) in which I assigned the value 15 to max_weight. This larger assignment, as I knew from vast experimentation with OTTER, would most likely cause OTTER to consume a great deal of memory and also cause far more CPU time to be required to finish the assignment. The explanation rests, as you have surmised, with the fact that the program would, with the larger max_weight, retain many, many more newly deduced items. That file differed in but one way, namely, the cited assigned value 15 (rather than 10) to max_weight. Indeed, so much more time and memory were required that only sixty-one theorems, of the sixty-three, were proved. In particular, HIF5 and H4441 were not proved, which, as noted earlier, did resist proof for a long time in the first arduous trip. Of the sixty-one theorems that were proved, OTTER found thirty-four shorter proofs (with 3b) than it had with 3a. Among the rather startling results, the program (with 3b) found a 21-step proof of the theorem denoted with 11 (whose name is 11 in the passive list), in contrast to a 43-step proof with 3a; the time required for the 21-step proof was approximately 135106 CPU-seconds, in contrast to approximately 82 CPU-seconds. For yet one more interesting comparison, the last theorem proved with 3b was H321 in approximately 1123161 CPU-seconds; with 3a, that same theorem was proved in approximately 5 CPU-seconds. With 3b, the proof of H321 was completed with retention of clause (6776552); with 3a, H321 was proved with retention of clause (2443). With 3a, 102 proofs were completed, including those that were not the shortest, which is what you can expect with the use of ancestor subsumption; with 3b, 168 were completed. In other words, with that powerful procedure, a program typically completes a proof and then, sometimes much later, finds a shorter proof, offering both to you. Although many, many experiments followed that relying on 3b, twenty-two proofs found with that input file remained as the shortest I ever found. Finding the final set of short proofs met with much resistance, as you will learn, in part because I (late in the game) became greedy, seeking to find (if possible) proofs for each of the sixty-three targets of length strictly less than 30 (deduced steps).

Now, at this point, I have decided to present the highlights, rather than detailing all that occurred. Of course, I almost certainly will discuss some of the attempts that yielded failure or little progress. For example, when I made but one change in 3b, assigning the value 12 to max_weight, with the goal of proving all sixty-three theorems in one run, with particular emphasis on the two not proved with 3b, those two were still not proved. So the run yielded little of interest for me.

Perhaps five days later, I asked myself what actions I could take in this proof-shortening expedition to produce in one run all sixty-three proofs, many of which would be shorter than I now had in hand. I chose to return to 3b, where the value assigned to max_weight was 15, but I adjoined various sets of resonators, each corresponding to a proof step taken from some short proof obtained with 3b and with 3a. My notion was to provide much guidance, in the form of a direction strategy, to perhaps enable OTTER to find proofs of all sixty-three theorems. At the same time, of course, the goal was to continue to find ever-shorter proofs. Well, I did find proofs of both HIF5 and H4441 as well as proofs of the other sixty-one; the two proofs of particular interest have lengths, respectively, of 63 and 48. However, although six of the proofs that were found, when compared with those found with 3b, were shorter, four were in fact longer. You thus are witnessing some of the complexity; indeed, I was certainly not interested in proofs getting longer.

The time seems right to skip here many of my experiments and concentrate on a goal that began to become evident to me. It was not totally evident for months. But the goal was to find sixty-three proofs none of them longer than length 29 (applications of condensed detachment). Of course, I recognized that such might not be possible; indeed, one or more of the sixty-three theorems might require at least 30 deduced steps, and perhaps far more. So, I shall in a manner of speaking look ahead, starting with what I had in hand at that time.

Of the sixty-three so-called short proofs, ten (in a file, for me, .out99t) were of length at least 30. the theorems whose proof length equaled or exceeded 30 were H423, H341, H4441, HIF5, HIF1, H351, H35, HIF2, H3422, and HIF4. I have listed them in the order they were completed. The shortest proof had length 31, and the longest had length 71. The theorem HIF4 was the evildoer that had a proof of length 71. These ten were found in less than 140 CPU-seconds. By way of adding to your understanding, at the time, in the middle of the more than five-month trip, I did not have explicitly in mind the discovery of sixty-three proofs of length strictly less than 30. But, not much later than that time, I did realize that was my goal. As you may have guessed, such a goal was in part motivated by my attachment to the decimal system, with the number 29 having more appeal, substantially more, than the number 30.

From the viewpoint of history, the ten that were resisting were in hand in roughly mid-February 2010. Although minor progress was made, it was not until early June that I had a run producing sixty-three proofs, with but six whose length was equal to or greater than 30. Those six, not too surprisingly, were among the ten just cited, namely, H3422, H351, H35, H423, HIF1, and HIF4. Their lengths were, respectively, 51, 39, 44, 34, 35, and 67. Of the sixty-three proofs found and just cited for February, twenty-two of the June proofs were shorter. Two improvements merit special mention. In the first, in contrast to the February proof for H4441 of length 48 and level 14, the June proof that was found was of length 11 and level 9. In the second, I now had, in place of a length 63 level 16 proof of HIF5, a length 20 level level 10 proof. Of course, you might naturally wonder what had transpired during those roughly four months (as I conducted an experiment called in99z21). After all, regardless of your major interest, the intent here is to provide at least some vague hints of how to conquer huge obstacles. Therefore, let us touch on the key actions.

In the June experiment, I assigned the value 20 (rather than 15) to max_weight, to permit OTTER to retain even more complex deductions than it had been obtaining in the so-called beginning. I believe a vital move, in the June experiment, concerned my choice of resonators to guide the search, formulas that had been shown to be most useful, implying that they themselves, or their functional shape, would positively influence the likelihood of progress. For the first set, I chose 37 proof steps from a much earlier (February) proof of HIF1; I next included 33 resonators from an earlier June experiment that proved HIF5; I finally included 305 sorted proof steps, from a slightly later June experiment. Then, of the ten Horn axioms, 6 and

7 and 8 (which focus mainly on the function *a*) were placed in list(usable), so they would not initiate applications of condensed detachment. The remaining seven (of the ten) axioms were placed in list(sos). I included no demodulators, equations to block the use of one or more formulas. In the same spirit, I assigned the value 7, rather than 6, to max_distinct_vars. But now, with this update complete, next in order is the attack I chose for finding proofs of length strictly less than 30 for the given six. I shall first focus on H3422, whose proof at the time was, as stated, of length 51 (applications of condensed detachment).

I have chosen to oversimplify the various attacks on the six because of the interplay and interwoven nature of my attempts to find the desired six proofs, each of length strictly less than 30. Nevertheless, the principles that underlie what I now offer capture the essence of the adventure. Quite a few experiments failed before the goal was reached. I relied on 303 resonators that corresponded to a sorting of proof steps that were obtained with the approach just described. I also included resonators corresponding to proofs so far obtained for H35 and HIF1, nicely illustrating the cited interwoven nature of the research under discussion. Of the ten Horn axioms, 6, 7, and 8 were placed in list(usable) so that the program would not use any of the three to initiate a line of reasoning. I assigned the value 20 to max_weight, 7 to max_distinct_vars, and 2 to pick_given_ratio. The motive for the first two assignments was to give OTTER extra room to consider complex formulas. The choice of 2 was to cause the program to choose, for inference-rule initiation, 2 formulas based on complexity, 1 based on first come first serve, 2, 1, and so on. As expected, I relied on the use of ancestor subsumption. I also instructed the program to focus with set(input_sos_first) on items in the initial set of support before focusing on newly retained items. After completing and presenting a 41-step proof of H3422, the program then offered a 28-step proof of level 12, found in less than 8 CPU-seconds. So one of the six had been conquered. For the curious, later I found a 26-step level-12 proof, which is the shortest I know of at this time in late 2010. You might find it challenging to discover a shorter proof.

Next for discussion is H351. In addition to the beginning-to-be-famous 303 resonators, I included 43 corresponding to a proof of H35 and 109 from a study that shortened a proof of HIF1-5. The experiment yielded three proofs of interest, of respective lengths 35, 34, and 28, the last of level 13. The study offered far more resistance than did H3422; indeed, the three proofs were completed in approximately 977, 1007, and 7315 CPU-seconds, respectively. Further, very many experiments under various conditions yielded insufficient progress of the type being sought. Later I found a 27-step level-12 proof, the shortest I currently know of.

As for H35, I followed the now-familiar path of trying one unsuccessful experiment after another. Finally, I produced an input file whose use quickly yielded a 30-step proof and, after more than 12385 CPU-seconds, presented me with a 27-step level-12 proof. This fine input file resembled somewhat that used to eventually find the proof of H351 of length less than 30. Among the resonators, however, in addition to the so-often used 303, I included resonators obtained from earlier studies of H35, studies that made progress but still fell short of the desired goal. Along the way, as I met huge resistance, I turned briefly to a level-saturation approach, an approach that did yield what turned out to be a crucial step, obtained after an expenditure of more than 892,138 CPU-seconds and the retention of a clause numbered 2974868. I also included resonators from a study of HIF1. Yes, I have found that, contrary to my total disinterest in level saturation evinced decades ago, such a search sometimes provides just what is needed for a sought-after shorter proof or for a first proof. After some additional experimentation, I had in hand a 25-step level-13 proof, and to this date I have found no shorter. I was, in the current exposition, halfway home, with three formulas yet to be proved, each with a proof of length strictly less than 30.

The formula H423 was conquered easily, with an approach that closely resembles that just described. The resonators used, I believe, played the key role, the usual 303, resonators from a mid-range proof of H35, those from an intermediate study of HIF4, and those from a 31-step proof of H423 itself. That 31-step proof was obtained, of course, by starting with the cited 34-step proof and keying on an intermediate proof of H3422 as well as other so-called intermediate proofs. (By intermediate I mean a proof whose length is short of the desired goal.) The desired proof that was offered is one of length 29 and level 15; I currently know of no shorter proof. One of the so pleasing features of OTTER is its offering the user the ability to seek proofs of many theorems in a single run, where the theorems may have little in common other than relying on a shared set of axioms.

You are now left with two formulas to learn about, HIF1 and HIF4. A glance at the early part of this notebook shows that, perhaps predictably, these two formulas presented a problem; they did even when seeking a first proof. Therefore, you might expect that seeking for each a proof of length strictly less than 30 might be a bit difficult. The approach I took again was quite like that discussed throughout in the context of the six whose proofs were, at one time, of length 30 or more. I relied on various sets of resonators that included the 303 used throughout this phase of my research, those corresponding to a 27-step proof of H35, those corresponding to a 32-step proof of HIF1, those corresponding to a 28-step proof of H351, and those corresponding to a 33-step proof of HIF4. Yes, you note that the proof of HIF4 that I eventually shortened and that in effect prompted in part this bit of narrative has length 67. Again, you have more evidence of the interwoven nature of this bit of research and of the fact that I took advantage of OTTER's being able to attack a number of theorems simultaneously. I also did modify the basic approach in some significant ways in the context of assigned values. In particular, because of the anticipated difficulty, I assigned 6 (rather than 7) to max_distinct_vars, believing that otherwise the program might never reach the goal. I also changed the value assigned to max_weight during the run, beginning with the value 20, but then reducing it to 8 after 1,500 items had been chosen to initiate applications of condensed detachment. Now, you might wonder about the small value of 8, being concerned that such a value might prevent OTTER from retaining a needed deduction. Well, keep in mind that a large number of resonators were being relied upon and that each was assigned the value 2 or smaller. Indeed, members of some of the sets were assigned the value -4, which caused the program to give an effective weight to newly deduced items a value less than 8.

With this input file, the program found proofs of respective lengths 32, 30, and 26, the last completed after approximately 42088 CPU-seconds. First you see that the task of finding a proof of HIF1 of length strictly less than 30 was indeed formidable, and you see how wonderfully robust OTTER is. Many experiments were required, or, at least, many were conducted. I eventually found a 19-step level-11 proof, and I have found no shorter. (For total clarity, once I found the proofs of length strictly less than 30, I did not make much of an attempt to find proofs of even shorter length.)

Finally, I turned my full attention on HIF4, but I did not focus on the cited 67-step proof found in out99z21. From what I can determine—but noting that my wandering through the huge number of experiments I conducted might have slightly misled me—I began in the following manner, with the treatment beginning more or less in the middle of a huge number of experiments. After some experimentation, I had in hand a 56-step proof of HIF4 (in out99z10), obtained by relying heavily on resonators taken from earlier runs that proved, among others, H3422, HIF5, H4441, and HIF1. The next highlight was obtaining a 48-step proof in a run that obtained proofs in the following order of length: 64, 63, 52, 51, 49, and 48. The 48-step proof was completed after approximately 16137 CPU-seconds and retention of clause (3195819). In the context of resonators, the main focus was on those from a proof of HIF5 (53 of them) and 305 shorter proof steps of other theorems proved earlier. After more experimentation, OTTER presented me with a 40-step proof, quickly found (out99z35). The key resonators were taken from 109 sorted proof steps of proofs of HIF1 through HIF5 and a 43-step proof of H35. Then, after more experiments, I finally had a proof in the thirties, one of length 34, completed in approximately 10271 CPU-seconds with retention of clause (3563234). Yes, you are witnessing a tortuous trip in search of the desired proof of HIF4, one of length strictly less than 30. This last breakthrough, producing the cited 34-step proof, merits a bit of discussion because it focuses on a technique I sometimes use when desperate.

The technique concerns the finding of key resonators, 28 of them. What I did was to take the last ten steps of the cited 40-step proof and place their negations in the passive list, with the goal of finding proofs of each. The important point was the fulfillment, if possible, of the discovery of a so-called shorter proof (than I had) of one of the ten. What McCune's program found for me (in out99z49) was first a 41-step proof but then a 28-step proof of the 38th step of the 40-step proof. Those proofs steps, 28 of them, were used as resonators in a run that presented me with the 34-step proof. I felt, at that time, that perhaps soon I might have a proof of length strictly less than 30. However, as history records the facts, more than a month of experiments was actually required, not constant experimentation but quite a bit of thought.

Attempts to use the 34-step proof as resonators produced little progress for weeks. However, after those weeks elapsed and I had a new idea, the corresponding run yielded proofs of respective lengths 34, 33, and 31. In order to obtain the 31-step proof, the key changes were, first, to assign the value 6 (rather than 7) to max_distinct_vars and 4 (rather than 2) to pick_given_ratio, and, second, to include what I believe were key resonators. The two key sets consisted of, first, eighteen resonators that corresponded to a proof of step 29 of a 33-step proof and, second, fifteen resonators from a somewhat different 33-step proof. The same day but a few hours later, I used the 31-step proof (as resonators) to obtain a 30-step proof. Approximately 1 hour later, with various sets of resonators including those corresponding to the just-cited 30-step proof and those corresponding to two proofs of H35 and a set corresponding to proofs that included proofs of HIF1 through HIF5, the lengthy and often frustrating journey had come to an end. In particular, in out99z55k, I found a 29-step proof. Indeed, for all sixty-three theorems, I possessed proofs each of which was of length strictly less than 30 (applications of condensed detachment). My perhaps unreasonable goal had been reached—so much for adherence to the decimal system.

So you have now completed a reading of two lengthy journeys, the second far longer than the first. I next turn to projects that might be of interest, as well as random notes that interest me and might interest you.

6. Challenges, Partial Review, and Random Notes

A challenge, if met, that could be of value in many ways focuses on automating much of the approach featured in the first journey. Equally, automating the search for elegant proofs in the context of length, the second journey, presents a challenge. A quite different challenge concerns finding proofs shorter than those cited, proofs that are included later in this notebook.

If you were to run the majority of the experiments I did, you would learn that the number of proofs for a given theorem is far greater than, I suspect, would be predicted even by experienced logicians or mathematicians. You would also discover that, from what might be termed a topological viewpoint, various types of proof can be and are found by OTTER. For example, some proofs, if mapped, look like a long line, with one step obtained from the preceding step considered with an even earlier step. for a much different type of proof, the last step sometimes is found by applying, in this case, condensed detachment to the preceding two steps. Further, such a proof might take the form of (in effect) two almost independent subproofs whose last steps, taken together, complete the desired total proof.

What I find appealing is that the use of a program such as OTTER enables you to find diverse types of proof and is not in general influenced the way an unaided researcher can be. As is well known, however, I view the use of a reasoning program as having access to a “tireless” assistant that can take advice, sometimes through the use of resonators, sometimes through the use of Veroff’s hints strategy, and sometimes in some other way. This tirelessness is exhibited by the large numbers I occasionally encountered, in the context of new formulas retained, new formulas deduced, and CPU time required to find the sought-after proof.

As the following large numbers show you, sometimes OTTER worked very hard to reach the desired goal. These numbers are even more impressive, to me, in that much strategy was used, strategy that sharply reduced the difficulty of reaching the target. At least one of the experiments deduced more than 1 billion conclusions before it terminated. Of that billion, more than 10 million were retained. (In at least one experiment, more than 796 hours were expended as measured by the wall clock, more than 2,000,000 CPU-seconds of computing. Further, in one of the experiments, more than 397,000 clauses were chosen to initiate inference-rule application.) As an example of some of the difficulty encountered, in one experiment to find an appropriate proof of H35, clause (3223082) was present in its proof, completed after 1596784 CPU-seconds of computing. Now, let me be clear, a drawn conclusion can be reached over and over again, just as many proofs can be found for certain theorems. Such redundancy is dealt with by merely deleting the extra copies, except in rare cases. Even in the experiment that yielded some useful resonators (taken from the file out9w), more than 895 million clauses were generated of which a bit more than 1 million were retained, exhibiting how much redundancy can occur and, even more, how many new deductions are discarded because the program found them too complex, namely, more than 822 million. I know of no way, a priori, to avoid deducing some conclusions many, many times. Hence, by removing duplicate conclusions, the procedures subsumption and back subsumption each play a vital role. In a vaguely related manner, you can use demodulation by, for example, having the program simplify and canonicalize deductions.

Although I, personally, cannot offer anything resembling an algorithm for finding a first proof or finding a more elegant proof, as repeatedly illustrated throughout this notebook, many approaches and so-called tricks can be used in either context. Indeed, you can, as I often do, include intermediate targets that, if reached, supply resonators for a succeeding run. And here you find yet another challenge. In particular, you could enhance your program, or OTTER, by enabling it to pause when intermediate targets have been reached—proofs of various lemmas you have chosen—and adjoin their proof steps as resonators or as hints, assigning to each a small value. If this enhancement were present, I would no doubt have more than occasionally relied upon it, sometimes avoiding hours of program inactivity as I slept or enjoyed some other pursuit. Yes, how exciting to return to a run and discover that it had paused, examined its results, and taken appropriate action.

Especially, but not exclusively, in the context of seeking a shorter proof, the following approach or trick has been of use. Imagine that you have been adjoining in a sequence of runs a set of demodulators, each included to block the use of some formula if and when deduced. Further, imagine that the sequence has led to finding shorter and ever shorter proofs, but a dead end has been reached. You might then, as I have done, replace the resonators in use with those corresponding to the latest "short" proof, remove the newly included demodulators, and see what occurs. Sometimes this approach results in yet additional advances; indeed, the newly adjoined demodulators can block too many paths, and their removal frees the program to follow paths not yet examined. As a related approach, you can so-to-speak back up a bit, focus on a somewhat longer proof than you have found, and then peruse a different path of reasoning.

Of the many tricks that can be used, a simple one to try, a trick that requires virtually no effort, is to take a run that yielded a proof of interest, but still short of your chosen target, and change the value assigned to pick_given_ratio. A smaller value than was used will have the program focus on retained deductions that are produced earlier and, quite likely, are rather complex; a larger value will emphasize for initiating applications of the inference rule items chosen based on complexity, rather than on when they were deduced. The idea is to cause the program to follow a quite different search path through the potentially huge space of deducible conclusions. (Coping with a huge space of deducible conclusions is an obstacle faced when seeking shorter and still sorter proofs, as it also is in seeking a first proof of a deep theorem.) Of course, you can achieve (in spirit) the same objective by a sharp change in the value assigned to max_weight and such, sometimes relying on what intuitively is a big change, say, from 20 to 28. How nice that, with a small twist of the wrist, you can initiate a possibly wildly new attack, which provides quite a contrast to what an unaided researcher might be able to do.

As the material in this notebook illustrates, I did enjoy pursuing this labyrinth—and enjoy similar ones—and journeys that sometimes present misleading clues. The various prizes won by this treasure hunt were, in some cases, unexpected. I found excitement in cases that lasted less than 6 CPU-seconds and cases that required more than 600 CPU-hours. Even though the theorems I studied here had already been proved in the literature, the approaches taken in this notebook (with the goal of finding proofs of each) have value in part because they illustrate how a program's power can be enhanced. As for seeking proofs shorter than offered in print, such an activity is reminiscent of trying to win a hard game or solve a difficult puzzle, where the effort can provide along the way useful clues, as well as some that are misleading.

Perhaps this notebook has suggested an interesting research topic. Fortunately, although the discussion focuses on OTTER, the ideas are general—they do not require the use of McCune's program. For example, since the subformula strategy proved useful in my experiments, the following variation, which you might call the dynamic subformula strategy, might prove intriguing. When a run is complete or terminated by you, you could take from the proofs of so-called intermediate targets, lemmas whose negation is placed in, say, list(passive), subformulas of their proof steps and adjoin those subformulas to be used in a succeeding run. You could assign small values to the new items and, possibly, based on them, recompute the weights of items already on the set of support list that had not yet been chosen as the focus of attention. Indeed, as you have seen, proof steps from one run can be used as powerful resonators in a later run; however, not unexpected, some newly included resonators can lead to shorter proofs for some theorems while, at the same time, lead to proofs longer than you have already found. The choice of which resonators to include and which to give lower or higher assigned values is an art indeed.

Perhaps you or some other energetic researcher will fulfill one of the goals I have occasionally contemplated. Specifically, you would take a textbook, apply the various approaches for finding shorter proofs, and produce a far simpler text containing the newer and shorter proofs. If you were fortunate, various lemmas that had been believed to be indispensable would now not be needed. Yes, I would like to hear of such a success. You certainly could satisfy another of the goals of automated reasoning, that of a thorough proofchecking of the entire text.

If you are seeking a first proof or seeking a proof shorter than you have so far discovered, you might try the *cramming strategy*. The simplest illustration of this strategy concerns the proof of a conjunction, say, of three targets. Consider you have a proof of two of the three in hand. You then place the proof steps in list(sos) and (in effect) attempt to force the program to use those steps on the way to finding a proof of the thir element. If you have, instead, a single target and, say, are seeking a shorter proof of it than you have found, you take a subproof of one of the so-called late steps in the proof in hand and cram or force, if it works, that subproof to be heavily used on the way to the desired shorter proof. The idea is to have the program use certain intermediate steps two, three, or more times in the so-called final proof. I successfully used in cramming in my focus on HIF4.

Before presenting the 63 "best" proofs I now have for the original 63 target theorems, I supply two proofs of HIF1. The first has length 35, and the second has length 19. You may find intrigue in determining why or how the 19-step proof could have been obtained from the 35-step proof. Less remote, but with a probability greater than zero, a study of the two proofs might lead to a new strategy to enhance the power of automated reasoning programs.

A 35-Step Proof of HIF1

```
----> UNIT CONFLICT at   1.54 sec ----> 10726 [binary,10725.1,75.1] $ANS(HIF1).
Length of proof is 35.  Level of proof is 11.

---------------- PROOF ----------------

1 [] -P(i(x,y))| -P(x)|P(y).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
7 [] P(i(k(x,y),x)).
8 [] P(i(k(x,y),y)).
9 [] P(i(i(x,y),i(i(x,z),i(x,k(y,z))))).
75 [] -P(k(i(k(i(p,q),i(p,r)),i(p,k(q,r))),i(i(p,k(q,r)),k(i(p,q),i(p,r)))))|$ANS(HIF1).
90 [hyper,1,6,6] P(i(i(i(x,i(y,z)),i(x,y)),i(i(x,i(y,z)),i(x,z)))).
91 [hyper,1,5,6] P(i(x,i(i(y,i(z,u)),i(i(y,z),i(y,u))))).
97 [hyper,1,5,7] P(i(x,i(k(y,z),y))).
102 [hyper,1,5,8] P(i(x,i(k(y,z),z))).
107 [hyper,1,5,9] P(i(x,i(i(y,z),i(i(y,u),i(y,k(z,u)))))).
111 [hyper,1,9,7] P(i(i(k(x,y),z),i(k(x,y),k(x,z)))).
149 [hyper,1,6,91] P(i(i(x,i(y,i(z,u))),i(x,i(i(y,z),i(y,u))))).
156 [hyper,1,5,97] P(i(x,i(y,i(k(z,u),z)))).
165 [hyper,1,5,102] P(i(x,i(y,i(k(z,u),u)))).
168 [hyper,1,6,107] P(i(i(x,i(y,z)),i(x,i(i(y,u),i(y,k(z,u)))))).
193 [hyper,1,5,111] P(i(x,i(i(k(y,z),u),i(k(y,z),k(y,u))))).
270 [hyper,1,149,5] P(i(i(x,y),i(i(z,x),i(z,y)))).
301 [hyper,1,90,156] P(i(i(x,i(i(k(y,z),y),u)),i(x,u))).
326 [hyper,1,90,165] P(i(i(x,i(i(k(y,z),z),u)),i(x,u))).
348 [hyper,1,6,193] P(i(i(x,i(k(y,z),u)),i(x,i(k(y,z),k(y,u))))).
455 [hyper,1,6,270] P(i(i(i(x,y),i(z,x)),i(i(x,y),i(z,y)))).
456 [hyper,1,5,270] P(i(x,i(i(y,z),i(i(u,y),i(u,z))))).
```

515 [hyper,1,301,326] P(i(i(i(k(x,y),x),i(i(k(z,u),u),v)),v)).
699 [hyper,1,348,5] P(i(x,i(k(y,z),k(y,x)))).
804 [hyper,1,455,107] P(i(i(i(i(x,y),i(x,k(z,y))),u),i(i(x,z),u))).
806 [hyper,1,455,97] P(i(i(x,y),i(k(x,z),y))).
814 [hyper,1,455,456] P(i(i(i(i(x,y),i(x,z)),u),i(i(y,z),u))).
1210 [hyper,1,6,699] P(i(i(x,k(y,z)),i(x,k(y,x)))).
1959 [hyper,1,149,806] P(i(i(x,i(y,z)),i(i(k(x,u),y),i(k(x,u),z)))).
2017 [hyper,1,804,814] P(i(i(i(x,y),z),i(i(y,u),i(i(x,y),k(z,i(x,u)))))).
3933 [hyper,1,326,1959] P(i(i(x,i(y,z)),i(k(x,y),z))).
3961 [hyper,1,814,2017] P(i(i(x,y),i(i(x,z),i(i(u,x),k(i(u,y),i(u,z)))))).
5832 [hyper,1,3933,9] P(i(k(i(x,y),i(x,z)),i(x,k(y,z)))).
5941 [hyper,1,515,3961] P(i(i(x,k(y,z)),k(i(x,y),i(x,z)))).
8164 [hyper,1,168,5] P(i(x,i(i(y,z),i(y,k(x,z))))).
8937 [hyper,1,6,8164] P(i(i(x,i(y,z)),i(x,i(y,k(x,z))))).
9628 [hyper,1,8937,107] P(i(x,i(i(y,z),k(x,i(i(y,u),i(y,k(z,u))))))).
10017 [hyper,1,9628,5832] P(i(i(x,y),k(i(k(i(z,u),i(z,v)),i(z,k(u,v))),i(i(x,w),i(x,k(y,w)))))).
10390 [hyper,1,1210,10017] P(i(i(x,y),k(i(k(i(z,u),i(z,v)),i(z,k(u,v))),i(x,y)))).
10725 [hyper,1,10390,5941] P(k(i(k(i(x,y),i(x,z)),i(x,k(y,z))),i(i(u,k(v,w)),k(i(u,v),i(u,w))))).

A 19-Step Proof of HIF1

----> UNIT CONFLICT at 24318.68 sec ----> 1847947 [binary,1847946.1,14.1] $ANS(HIF1).
Length of proof is 19. Level of proof is 11.

---------------- PROOF ----------------

1 [] -P(i(x,y))| -P(x)|P(y).
4 [] P(i(x,i(y,x))).
5 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
6 [] P(i(k(x,y),x)).
7 [] P(i(k(x,y),y)).
8 [] P(i(i(x,y),i(i(x,z),i(x,k(y,z))))).
14 [] -P(k(i(k(i(p,q),i(p,r)),i(p,k(q,r))),i(i(p,k(q,r)),k(i(p,q),i(p,r)))))|$ANS(HIF1).
31 [hyper,1,4,5] P(i(x,i(i(y,i(z,u)),i(i(y,z),i(y,u))))).
32 [hyper,1,5,4] P(i(i(x,y),i(x,x))).
36 [hyper,1,4,6] P(i(x,i(k(y,z),y))).
41 [hyper,1,5,8] P(i(i(i(x,y),i(x,z)),i(i(x,y),i(x,k(y,z))))).
88 [hyper,1,5,31] P(i(i(x,i(y,i(z,u))),i(x,i(i(y,z),i(y,u))))).
109 [hyper,1,41,32] P(i(i(x,y),i(x,k(y,x)))).
178 [hyper,1,88,4] P(i(i(x,y),i(i(z,x),i(z,y)))).
267 [hyper,1,5,178] P(i(i(i(x,y),i(z,x)),i(i(x,y),i(z,y)))).
281 [hyper,1,178,7] P(i(i(x,k(y,z)),i(x,z))).
1060 [hyper,1,267,36] P(i(i(x,y),i(k(x,z),y))).
1074 [hyper,1,5,36] P(i(i(x,k(y,z)),i(x,y))).
1855 [hyper,1,88,1060] P(i(i(x,i(y,z)),i(i(k(x,u),y),i(k(x,u),z)))).
2726 [hyper,1,1855,8] P(i(i(k(i(x,y),z),i(x,u)),i(k(i(x,y),z),i(x,k(y,u))))).
7397 [hyper,1,8,1074] P(i(i(i(x,k(y,z)),u),i(i(x,k(y,z)),k(i(x,y),u)))).
7654 [hyper,1,2726,7] P(i(k(i(x,y),i(x,z)),i(x,k(y,z)))).
11126 [hyper,1,7397,281] P(i(i(x,k(y,z)),k(i(x,y),i(x,z)))).
13457 [hyper,1,4,7654] P(i(x,i(k(i(y,z),i(y,u)),i(y,k(z,u))))).
1847936 [hyper,1,109,13457] P(i(x,k(i(k(i(y,z),i(y,u)),i(y,k(z,u))),x))).
1847946 [hyper,1,1847936,11126] P(k(i(k(i(x,y),i(x,z)),i(x,k(y,z))),i(i(u,k(v,w)),k(i(u,v),i(u,w))))).

7. Some Short Proofs

At this point, I supply the proofs of the 63 theorems of interest here, the shortest I know of at this time in early 2011.

Proof of 01, .out99r.

```
----> UNIT CONFLICT at   0.07 sec ----> 609 [binary,608.1,12.1] $ANS(01).

Length of proof is 5.  Level of proof is 3.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
9 [] P(i(i(x,y),i(i(x,z),i(x,k(y,z))))).
12 [] -P(i(b,k(b,b))) | $ANS(01).
92 [hyper,1,6,5] P(i(i(x,y),i(x,x))).
103 [hyper,1,6,9] P(i(i(i(x,y),i(x,z)),i(i(x,y),i(x,k(y,z))))).
162 [hyper,1,92,5] P(i(x,x)).
204 [hyper,1,103,92] P(i(i(x,y),i(x,k(y,x)))).
608 [hyper,1,204,162] P(i(x,k(x,x))).
```

Proof of 02, .out99r.

```
----> UNIT CONFLICT at   0.03 sec ----> 222 [binary,221.1,13.1] $ANS(02).

Length of proof is 2.  Level of proof is 2.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
7 [] P(i(k(x,y),x)).
8 [] P(i(k(x,y),y)).
9 [] P(i(i(x,y),i(i(x,z),i(x,k(y,z))))).
13 [] -P(i(k(b,c),k(c,b))) | $ANS(02).
105 [hyper,1,9,8] P(i(i(k(x,y),z),i(k(x,y),k(y,z)))).
221 [hyper,1,105,7] P(i(k(x,y),k(y,x))).
```

Proof of 03, .out99z21.

```
----> UNIT CONFLICT at   0.27 sec ----> 2255 [binary,2254.1,14.1] $ANS(03).

Length of proof is 13.  Level of proof is 6.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
7 [] P(i(k(x,y),x)).
```

8 [] P(i(k(x,y),y)).
9 [] P(i(i(x,y),i(i(x,z),i(x,k(y,z))))).
14 [] -P(i(i(b,c),i(k(b,d),k(c,d)))) | $ANS(03).
90 [hyper,1,6,6] P(i(i(i(x,i(y,z)),i(x,y)),i(i(x,i(y,z)),i(x,z)))).
91 [hyper,1,5,6] P(i(x,i(i(y,i(z,u)),i(i(y,z),i(y,u))))).
97 [hyper,1,5,7] P(i(x,i(k(y,z),y))).
102 [hyper,1,5,8] P(i(x,i(k(y,z),z))).
149 [hyper,1,6,91] P(i(i(x,i(y,i(z,u))),i(x,i(i(y,z),i(y,u))))).
165 [hyper,1,5,102] P(i(x,i(y,i(k(z,u),u)))).
270 [hyper,1,149,5] P(i(i(x,y),i(i(z,x),i(z,y)))).
326 [hyper,1,90,165] P(i(i(x,i(i(k(y,z),z),u)),i(x,u))).
455 [hyper,1,6,270] P(i(i(i(x,y),i(z,x)),i(i(x,y),i(z,y)))).
538 [hyper,1,326,9] P(i(i(k(x,y),z),i(k(x,y),k(z,y)))).
806 [hyper,1,455,97] P(i(i(x,y),i(k(x,z),y))).
935 [hyper,1,270,538] P(i(i(x,i(k(y,z),u)),i(x,i(k(y,z),k(u,z))))).
2254 [hyper,1,935,806] P(i(i(x,y),i(k(x,z),k(y,z)))).

Proof of 04, .out99z66.

----> UNIT CONFLICT at 0.04 sec ----> 250 [binary,249.1,24.1] $ANS(H221).

Length of proof is 4. Level of proof is 3.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
7 [] P(i(k(x,y),x)).
24 [] -P(i(p,p)) | $ANS(H221).
103 [hyper,1,6,5] P(i(i(x,y),i(x,x))).
106 [hyper,1,5,7] P(i(x,i(k(y,z),y))).
162 [hyper,1,5,106] P(i(x,i(y,i(k(z,u),z)))).
249 [hyper,1,103,162] P(i(x,x)).

Proof of 06, .out99r.

----> UNIT CONFLICT at 0.02 sec ----> 181 [binary,180.1,16.1] $ANS(06).

Length of proof is 2. Level of proof is 2.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
7 [] P(i(k(x,y),x)).
8 [] P(i(k(x,y),y)).
16 [] -P(i(k(b,i(b,c)),c)) | $ANS(06).
98 [hyper,1,6,8] P(i(i(k(x,i(y,z)),y),i(k(x,i(y,z)),z))).
180 [hyper,1,98,7] P(i(k(x,i(x,y)),y)).

Proof of 08, .out99r.

----> UNIT CONFLICT at   0.05 sec ----> 452 [binary,451.1,17.1] $ANS(08).

Length of proof is 4. Level of proof is 3.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
2 [] P(i(x,a(x,y))).
3 [] P(i(x,a(y,x))).
4 [] P(i(i(x,y),i(i(z,y),i(a(x,z),y)))).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
17 [] -P(i(a(b,c),a(c,b))) | $ANS(08).
89 [hyper,1,5,2] P(i(x,i(y,a(y,z)))).
93 [hyper,1,6,4] P(i(i(i(x,y),i(z,y)),i(i(x,y),i(a(x,z),y)))).
165 [hyper,1,93,89] P(i(i(x,a(y,z)),i(a(x,y),a(y,z)))).
451 [hyper,1,165,3] P(i(a(x,y),a(y,x))).

Proof of 09, .out99z43.

----> UNIT CONFLICT at 48381.81 sec ----> 3824295 [binary,3824294.1,18.1] $ANS(09).

Length of proof is 5. Level of proof is 5.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
4 [] P(i(i(x,y),i(i(z,y),i(a(x,z),y)))).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
7 [] P(i(k(x,y),x)).
8 [] P(i(k(x,y),y)).
18 [] -P(i(k(i(b,d),i(c,d)),i(a(b,c),d))) | $ANS(09).
94 [hyper,1,5,4] P(i(x,i(i(y,z),i(i(u,z),i(a(y,u),z))))).
219 [hyper,1,6,94] P(i(i(x,i(y,z)),i(x,i(i(u,z),i(a(y,u),z))))).
989 [hyper,1,219,7] P(i(k(i(x,y),z),i(i(u,y),i(a(x,u),y)))).
2990 [hyper,1,6,989] P(i(i(k(i(x,y),z),i(u,y)),i(k(i(x,y),z),i(a(x,u),y)))).
3824294 [hyper,1,2990,8] P(i(k(i(x,y),i(z,y)),i(a(x,z),y))).

Proof of 11, .out99z5.

----> UNIT CONFLICT at 19.75 sec ----> 37050 [binary,37049.1,19.1] $ANS(11).

Length of proof is 15. Level of proof is 11.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).

7 [] P(i(k(x,y),x)).
8 [] P(i(k(x,y),y)).
10 [] P(i(i(x,n(y)),i(y,n(x)))).
11 [] P(i(n(x),i(x,y))).
19 [] -P(i(k(i(b,c),i(b,n(c))),n(b))) | $ANS(11).
96 [hyper,1,6,6] P(i(i(i(x,i(y,z)),i(x,y)),i(i(x,i(y,z)),i(x,z)))).
97 [hyper,1,5,6] P(i(x,i(i(y,i(z,u)),i(i(y,z),i(y,u))))).
99 [hyper,1,5,7] P(i(x,i(k(y,z),y))).
133 [hyper,1,6,97] P(i(i(x,i(y,i(z,u))),i(x,i(i(y,z),i(y,u))))).
134 [hyper,1,96,99] P(i(i(k(x,y),i(x,z)),i(k(x,y),z))).
182 [hyper,1,133,5] P(i(i(x,y),i(i(z,x),i(z,y)))).
229 [hyper,1,182,182] P(i(i(x,i(y,z)),i(x,i(i(u,y),i(u,z))))).
236 [hyper,1,182,10] P(i(i(x,i(y,n(z))),i(x,i(z,n(y))))).
314 [hyper,1,229,182] P(i(i(x,y),i(i(z,i(u,x)),i(z,i(u,y))))).
428 [hyper,1,314,11] P(i(i(x,i(y,n(z))),i(x,i(y,i(z,u))))).
3573 [hyper,1,428,8] P(i(k(x,i(y,n(z))),i(y,i(z,u)))).
10133 [hyper,1,133,3573] P(i(k(x,i(y,n(z))),i(i(y,z),i(y,u)))).
19605 [hyper,1,134,10133] P(i(k(i(x,y),i(x,n(y))),i(x,z))).
28450 [hyper,1,236,19605] P(i(k(i(x,y),i(x,n(y))),i(z,n(x)))).
37049 [hyper,1,134,28450] P(i(k(i(x,y),i(x,n(y))),n(x))).

Proof of H2, .out99r.

----> UNIT CONFLICT at 0.05 sec ----> 407 [binary,405.1,20.1] $ANS(H2).

Length of proof is 3. Level of proof is 3.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
20 [] -P(i(i(q,r),i(i(p,q),i(p,r)))) | $ANS(H2).
91 [hyper,1,5,6] P(i(x,i(i(y,i(z,u)),i(i(y,z),i(y,u))))).
153 [hyper,1,6,91] P(i(i(x,i(y,i(z,u))),i(x,i(i(y,z),i(y,u))))).
405 [hyper,1,153,5] P(i(i(x,y),i(i(z,x),i(z,y)))).

Proof of H3, .out99r.

----> UNIT CONFLICT at 1121869.81 sec ----> 6770981 [binary,6770980.1,21.1] $ANS(H3).

Length of proof is 8. Level of proof is 7.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
21 [] -P(i(i(p,i(q,r)),i(q,i(p,r)))) | ANS(H3).
83 [hyper,1,5,5] P(i(x,i(y,i(z,y)))).
90 [hyper,1,6,6] P(i(i(i(x,i(y,z)),i(x,y)),i(i(x,i(y,z)),i(x,z)))).
152 [hyper,1,90,83] P(i(i(x,i(i(y,x),z)),i(x,z))).

398 [hyper,1,5,152] P(i(x,i(i(y,i(i(z,y),u)),i(y,u)))).
14818 [hyper,1,152,398] P(i(i(i(x,y),z),i(y,z))).
176768 [hyper,1,5,14818] P(i(x,i(i(i(y,z),u),i(z,u)))).
1231934 [hyper,1,6,176768] P(i(i(x,i(i(y,z),u)),i(x,i(z,u)))).
6770980 [hyper,1,1231934,6] P(i(i(x,i(y,z)),i(y,i(x,z)))).

Proof of N1, .out99z27.

----> UNIT CONFLICT at 1841.40 sec ----> 1065560 [binary,1065559.1,22.1] $ANS(N1).

Length of proof is 7. Level of proof is 7.

---------------- PROOF ----------------

1 [] -P(i(x,y))| -P(x)|P(y).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
10 [] P(i(i(x,n(y)),i(y,n(x)))).
22 [] -P(i(i(p,n(p)),n(p)))|$ANS(N1).
116 [hyper,1,6,10] P(i(i(i(x,n(y)),y),i(i(x,n(y)),n(x)))).
1432 [hyper,1,5,116] P(i(x,i(i(i(y,n(z)),z),i(i(y,n(z)),n(y))))).
170761 [hyper,1,6,1432] P(i(i(x,i(i(y,n(z)),z)),i(x,i(i(y,n(z)),n(y))))).
491806 [hyper,1,170761,5] P(i(x,i(i(y,n(x)),n(y)))).
702136 [hyper,1,6,491806] P(i(i(x,i(y,n(x))),i(x,n(y)))).
914010 [hyper,1,702136,491806] P(i(x,n(i(x,n(x))))).
1065559 [hyper,1,10,914010] P(i(i(x,n(x)),n(x))).

Proof of N2, .out99r.

----> UNIT CONFLICT at 6766.08 sec ----> 1402239 [binary,1402238.1,23.1] $ANS(N2).

Length of proof is 4. Level of proof is 4.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
11 [] P(i(n(x),i(x,y))).
23 [] -P(i(p,i(n(p),q))) | $ANS(N2).
117 [hyper,1,6,11] P(i(i(n(x),x),i(n(x),y))).
805 [hyper,1,5,117] P(i(x,i(i(n(y),y),i(n(y),z)))).
37897 [hyper,1,6,805] P(i(i(x,i(n(y),y)),i(x,i(n(y),z)))).
1402238 [hyper,1,37897,5] P(i(x,i(n(x),y))).

Proof of H221, .out99rw.

----> UNIT CONFLICT at 0.02 sec ----> 161 [binary,160.1,24.1] $ANS(H221).

Length of proof is 3. Level of proof is 2.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
24 [] -P(i(p,p)) | $ANS(H221).
91 [hyper,1,5,6] P(i(x,i(i(y,i(z,u)),i(i(y,z),i(y,u))))).
92 [hyper,1,6,5] P(i(i(x,y),i(x,x))).
160 [hyper,1,92,91] P(i(x,x)).

Proof of H223, .out99z21.

----> UNIT CONFLICT at 292.50 sec ----> 343748 [binary,343747.1,25.1] $ANS(H223).

Length of proof is 22. Level of proof is 10.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
7 [] P(i(k(x,y),x)).
8 [] P(i(k(x,y),y)).
9 [] P(i(i(x,y),i(i(x,z),i(x,k(y,z))))).
25 [] -P(i(k(i(p,q),i(r,d)),i(k(p,r),k(q,d)))) | $ANS(H223).
90 [hyper,1,6,6] P(i(i(i(x,i(y,z)),i(x,y)),i(i(x,i(y,z)),i(x,z)))).
91 [hyper,1,5,6] P(i(x,i(i(y,i(z,u)),i(i(y,z),i(y,u))))).
97 [hyper,1,5,7] P(i(x,i(k(y,z),y))).
102 [hyper,1,5,8] P(i(x,i(k(y,z),z))).
107 [hyper,1,5,9] P(i(x,i(i(y,z),i(i(y,u),i(y,k(z,u)))))).
149 [hyper,1,6,91] P(i(i(x,i(y,i(z,u))),i(x,i(i(y,z),i(y,u))))).
165 [hyper,1,5,102] P(i(x,i(y,i(k(z,u),u)))).
270 [hyper,1,149,5] P(i(i(x,y),i(i(z,x),i(z,y)))).
326 [hyper,1,90,165] P(i(i(x,i(i(k(y,z),z),u)),i(x,u))).
455 [hyper,1,6,270] P(i(i(i(x,y),i(z,x)),i(i(x,y),i(z,y)))).
790 [hyper,1,270,455] P(i(i(x,i(i(y,z),i(u,y))),i(x,i(i(y,z),i(u,z))))).
804 [hyper,1,455,107] P(i(i(i(i(x,y),i(x,k(z,y))),u),i(i(x,z),u))).
805 [hyper,1,455,102] P(i(i(x,y),i(k(z,x),y))).
806 [hyper,1,455,97] P(i(i(x,y),i(k(x,z),y))).
1959 [hyper,1,149,806] P(i(i(x,i(y,z)),i(i(k(x,u),y),i(k(x,u),z)))).
3933 [hyper,1,326,1959] P(i(i(x,i(y,z)),i(k(x,y),z))).
79961 [hyper,1,790,5] P(i(i(x,y),i(i(y,z),i(x,z)))).
80416 [hyper,1,79961,806] P(i(i(i(k(x,y),z),u),i(i(x,z),u))).
80417 [hyper,1,79961,805] P(i(i(i(k(x,y),z),u),i(i(y,z),u))).
147709 [hyper,1,804,80417] P(i(i(k(x,y),z),i(i(y,u),i(k(x,y),k(z,u))))).
236253 [hyper,1,80416,147709] P(i(i(x,y),i(i(z,u),i(k(x,z),k(y,u))))).
343747 [hyper,1,3933,236253] P(i(k(i(x,y),i(z,u)),i(k(x,z),k(y,u)))).

Proof of H225, .out99z17.

----> UNIT CONFLICT at 16061.85 sec ----> 3829974 [binary,3829973.1,27.1] $ANS(H225).

Length of proof is 8. Level of proof is 6.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
7 [] P(i(k(x,y),x)).
8 [] P(i(k(x,y),y)).
9 [] P(i(i(x,y),i(i(x,z),i(x,k(y,z))))).
27 [] -P(i(k(p,i(q,r)),i(q,k(p,r)))) | $ANS(H225).
83 [hyper,1,5,5] P(i(x,i(y,i(z,y)))).
107 [hyper,1,5,9] P(i(x,i(i(y,z),i(i(y,u),i(y,k(z,u)))))).
130 [hyper,1,6,83] P(i(i(x,y),i(x,i(z,y)))).
230 [hyper,1,6,107] P(i(i(x,i(y,z)),i(x,i(i(y,u),i(y,k(z,u)))))).
424 [hyper,1,130,7] P(i(k(x,y),i(z,x))).
1794 [hyper,1,230,424] P(i(k(x,y),i(i(z,u),i(z,k(x,u))))).
6035 [hyper,1,6,1794] P(i(i(k(x,y),i(z,u)),i(k(x,y),i(z,k(x,u))))).
3829973 [hyper,1,6035,8] P(i(k(x,i(y,z)),i(y,k(x,z)))).

Proof of H226, .out99r.

----> UNIT CONFLICT at 554.85 sec ----> 451447 [binary,451446.1,28.1] $ANS(H226).

Length of proof is 6. Level of proof is 6.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
9 [] P(i(i(x,y),i(i(x,z),i(x,k(y,z))))).
28 [] -P(i(p,i(q,k(q,p)))) | $ANS(H226).
92 [hyper,1,6,5] P(i(i(x,y),i(x,x))).
162 [hyper,1,92,5] P(i(x,x)).
444 [hyper,1,9,162] P(i(i(x,y),i(x,k(x,y)))).
2010 [hyper,1,5,444] P(i(x,i(i(y,z),i(y,k(y,z))))).
4873 [hyper,1,6,2010] P(i(i(x,i(y,z)),i(x,i(y,k(y,z))))).
451446 [hyper,1,4873,5] P(i(x,i(y,k(y,x)))).

Proof of H228, .out99z21.

----> UNIT CONFLICT at   0.10 sec ----> 807 [binary,806.1,31.1] $ANS(H228).

Length of proof is 6. Level of proof is 5.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
7 [] P(i(k(x,y),x)).

31 [] -P(i(i(p,c),i(k(p,b),c))) | $ANS(H228).
91 [hyper,1,5,6] P(i(x,i(i(y,i(z,u)),i(i(y,z),i(y,u))))).
97 [hyper,1,5,7] P(i(x,i(k(y,z),y))).
149 [hyper,1,6,91] P(i(i(x,i(y,i(z,u))),i(x,i(i(y,z),i(y,u))))).
270 [hyper,1,149,5] P(i(i(x,y),i(i(z,x),i(z,y)))).
455 [hyper,1,6,270] P(i(i(i(x,y),i(z,x)),i(i(x,y),i(z,y)))).
806 [hyper,1,455,97] P(i(i(x,y),i(k(x,z),y))).

Proof of H2281, .out99r.

----> UNIT CONFLICT at 0.01 sec ----> 124 [binary,123.1,32.1] $ANS(H2281).

Length of proof is 2. Level of proof is 2.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
32 [] -P(i(i(p,b),i(p,i(c,b)))) | $ANS(H2281).
83 [hyper,1,5,5] P(i(x,i(y,i(z,y)))).
123 [hyper,1,6,83] P(i(i(x,y),i(x,i(z,y)))).

Proof of H2282, .out99z66.

----> UNIT CONFLICT at 28232.67 sec ----> 3769806 [binary,3769805.1,33.1] $ANS(H2282).

Length of proof is 13. Level of proof is 9.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
7 [] P(i(k(x,y),x)).
8 [] P(i(k(x,y),y)).
9 [] P(i(i(x,y),i(i(x,z),i(x,k(y,z))))).
33 [] -P(i(k(p,i(k(p,b),c)),i(b,c))) | $ANS(H2282).
100 [hyper,1,5,6] P(i(x,i(i(y,i(z,u)),i(i(y,z),i(y,u))))).
119 [hyper,1,9,7] P(i(i(k(x,y),z),i(k(x,y),k(x,z)))).
143 [hyper,1,6,100] P(i(i(x,i(y,i(z,u))),i(x,i(i(y,z),i(y,u))))).
225 [hyper,1,143,5] P(i(i(x,y),i(i(z,x),i(z,y)))).
293 [hyper,1,225,225] P(i(i(x,i(y,z)),i(x,i(i(u,y),i(u,z))))).
302 [hyper,1,225,119] P(i(i(x,i(k(y,z),u)),i(x,i(k(y,z),k(y,u))))).
413 [hyper,1,143,293] P(i(i(x,i(y,z)),i(i(x,i(u,y)),i(x,i(u,z))))).
510 [hyper,1,302,5] P(i(x,i(k(y,z),k(y,x)))).
1364 [hyper,1,6,510] P(i(i(x,k(y,z)),i(x,k(y,x)))).
2167 [hyper,1,225,1364] P(i(i(x,i(y,k(z,u))),i(x,i(y,k(z,y))))).
17739 [hyper,1,413,8] P(i(i(k(x,i(y,z)),i(u,y)),i(k(x,i(y,z)),i(u,z)))).
275540 [hyper,1,2167,5] P(i(k(x,y),i(z,k(x,z)))).
3769805 [hyper,1,17739,275540] P(i(k(x,i(k(x,y),z)),i(y,z))).

Proof of H229, .out99z21.

----> UNIT CONFLICT at 23.27 sec ----> 79962 [binary,79961.1,34.1] $ANS(H229).

Length of proof is 6. Level of proof is 6.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
34 [] -P(i(i(p,b),i(i(b,c),i(p,c)))) | $ANS(H229).
91 [hyper,1,5,6] P(i(x,i(i(y,i(z,u)),i(i(y,z),i(y,u))))).
149 [hyper,1,6,91] P(i(i(x,i(y,i(z,u))),i(x,i(i(y,z),i(y,u))))).
270 [hyper,1,149,5] P(i(i(x,y),i(i(z,x),i(z,y)))).
455 [hyper,1,6,270] P(i(i(i(x,y),i(z,x)),i(i(x,y),i(z,y)))).
790 [hyper,1,270,455] P(i(i(x,i(i(y,z),i(u,y))),i(x,i(i(y,z),i(u,z))))).
79961 [hyper,1,790,5] P(i(i(x,y),i(i(y,z),i(x,z)))).

Proof of H2291, .out99r.

----> UNIT CONFLICT at   0.05 sec ----> 406 [binary,405.1,35.1] $ANS(H2291).

Length of proof is 3. Level of proof is 3.

---------------- PROOF ----------------

1 [] -P(i(x,y)) -P(x) | P(y).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
35 [] -P(i(i(b,c),i(i(p,b),i(p,c)))) | $ANS(H2291).
91 [hyper,1,5,6] P(i(x,i(i(y,i(z,u)),i(i(y,z),i(y,u))))).
153 [hyper,1,6,91] P(i(i(x,i(y,i(z,u))),i(x,i(i(y,z),i(y,u))))).
405 [hyper,1,153,5] P(i(i(x,y),i(i(z,x),i(z,y)))).

Proof of H23, .out99z27.

----> UNIT CONFLICT at 11596.80 sec ----> 3101660 [binary,3101659.1,36.1] $ANS(H23).

Length of proof is 13. Level of proof is 7.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
7 [] P(i(k(x,y),x)).
8 [] P(i(k(x,y),y)).
9 [] P(i(i(x,y),i(i(x,z),i(x,k(y,z))))).
36 [] -P(i(k(k(p,b),c),k(p,k(b,c)))) | $ANS(H23).
83 [hyper,1,5,5] P(i(x,i(y,i(z,y)))).
110 [hyper,1,9,8] P(i(i(k(x,y),z),i(k(x,y),k(y,z)))).

111 [hyper,1,9,7] P(i(i(k(x,y),z),i(k(x,y),k(x,z)))).
130 [hyper,1,6,83] P(i(i(x,y),i(x,i(z,y)))).
219 [hyper,1,5,110] P(i(x,i(i(k(y,z),u),i(k(y,z),k(z,u))))).
239 [hyper,1,5,111] P(i(x,i(i(k(y,z),u),i(k(y,z),k(y,u))))).
310 [hyper,1,130,8] P(i(k(x,y),i(z,y))).
655 [hyper,1,6,219] P(i(i(x,i(k(y,z),u)),i(x,i(k(y,z),k(z,u))))).
674 [hyper,1,6,239] P(i(i(x,i(k(y,z),u)),i(x,i(k(y,z),k(y,u))))).
1566 [hyper,1,655,310] P(i(k(x,y),i(k(z,u),k(u,y)))).
3388 [hyper,1,674,1566] P(i(k(x,y),i(k(z,u),k(z,k(u,y))))).
7727 [hyper,1,6,3388] P(i(i(k(x,y),k(z,u)),i(k(x,y),k(z,k(u,y))))).
3101659 [hyper,1,7727,7] P(i(k(k(x,y),z),k(x,k(y,z)))).

Proof of H231, .out99z21.

----> UNIT CONFLICT at 21.29 sec ----> 74924 [binary,74923.1,37.1] $ANS(H231).

Length of proof is 7. Level of proof is 7.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
7 [] P(i(k(x,y),x)).
8 [] P(i(k(x,y),y)).
9 [] P(i(i(x,y),i(i(x,z),i(x,k(y,z))))).
37 [] -P(i(k(k(p,b),c),k(k(b,p),c))) | $ANS(H231).
110 [hyper,1,9,8] P(i(i(k(x,y),z),i(k(x,y),k(y,z)))).
186 [hyper,1,110,7] P(i(k(x,y),k(y,x))).
9703 [hyper,1,5,186] P(i(x,i(k(y,z),k(z,y)))).
10161 [hyper,1,6,9703] P(i(i(x,k(y,z)),i(x,k(z,y)))).
10533 [hyper,1,10161,7] P(i(k(k(x,y),z),k(y,x))).
34687 [hyper,1,110,10533] P(i(k(k(x,y),z),k(z,k(y,x)))).
74923 [hyper,1,10161,34687] P(i(k(k(x,y),z),k(k(y,x),z))).

Proof of H232, .out99z21.

----> UNIT CONFLICT at 47.18 sec ----> 121346 [binary,121345.1,38.1] $ANS(H232).

Length of proof is 10. Level of proof is 6.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
7 [] P(i(k(x,y),x)).
8 [] P(i(k(x,y),y)).
9 [] P(i(i(x,y),i(i(x,z),i(x,k(y,z))))).
38 [] -P(i(k(p,k(b,c)),k(k(p,b),c))) | $ANS(H232).
97 [hyper,1,5,7] P(i(x,i(k(y,z),y))).
102 [hyper,1,5,8] P(i(x,i(k(y,z),z))).

111 [hyper,1,9,7] P(i(i(k(x,y),z),i(k(x,y),k(x,z)))).
155 [hyper,1,6,97] P(i(i(x,k(y,z)),i(x,y))).
164 [hyper,1,6,102] P(i(i(x,k(y,z)),i(x,z))).
6877 [hyper,1,155,8] P(i(k(x,k(y,z)),y)).
7833 [hyper,1,164,8] P(i(k(x,k(y,z)),z)).
31272 [hyper,1,111,6877] P(i(k(x,k(y,z)),k(x,y))).
68938 [hyper,1,9,31272] P(i(i(k(x,k(y,z)),u),i(k(x,k(y,z)),k(k(x,y),u)))).
121345 [hyper,1,68938,7833] P(i(k(x,k(y,z)),k(k(x,y),z))).

Proof of H24, .out99z21.

----> UNIT CONFLICT at 50.46 sec ----> 126533 [binary,126532.1,39.1] $ANS(H24).

Length of proof is 9. Level of proof is 6.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
7 [] P(i(k(x,y),x)).
8 [] P(i(k(x,y),y)).
39 [] -P(i(k(k(i(p,b),k(b,c)),i(c,d)),i(p,d))) | $ANS(H24).
83 [hyper,1,5,5] P(i(x,i(y,i(z,y)))).
101 [hyper,1,6,8] P(i(i(k(x,i(y,z)),y),i(k(x,i(y,z)),z))).
102 [hyper,1,5,8] P(i(x,i(k(y,z),z))).
130 [hyper,1,6,83] P(i(i(x,y),i(x,i(z,y)))).
164 [hyper,1,6,102] P(i(i(x,k(y,z)),i(x,z))).
7834 [hyper,1,164,7] P(i(k(k(x,y),z),y)).
32984 [hyper,1,164,7834] P(i(k(k(x,k(y,z)),u),z)).
70597 [hyper,1,101,32984] P(i(k(k(x,k(y,z)),i(z,u)),u)).
126532 [hyper,1,130,70597] P(i(k(k(x,k(y,z)),i(z,u)),i(v,u))).

Proof of H32, .out99z31.

----> UNIT CONFLICT at 25699.59 sec ----> 4968947 [binary,4968946.1,40.1] $ANS(H32).

Length of proof is 22. Level of proof is 8.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
2 [] P(i(x,a(x,y))).
3 [] P(i(x,a(y,x))).
4 [] P(i(i(x,y),i(i(z,y),i(a(x,z),y)))).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
40 [] -P(i(a(a(p,b),c),a(p,a(b,c)))) | $ANS(H32).
87 [hyper,1,5,4] P(i(x,i(i(y,z),i(i(u,z),i(a(y,u),z))))).
88 [hyper,1,5,3] P(i(x,i(y,a(z,y)))).
89 [hyper,1,5,2] P(i(x,i(y,a(y,z)))).
91 [hyper,1,5,6] P(i(x,i(i(y,i(z,u)),i(i(y,z),i(y,u))))).

95 [hyper,1,6,4] P(i(i(i(x,y),i(z,y)),i(i(x,y),i(a(x,z),y)))).
136 [hyper,1,6,88] P(i(i(x,y),i(x,a(z,y)))).
164 [hyper,1,6,91] P(i(i(x,i(y,i(z,u))),i(x,i(i(y,z),i(y,u))))).
188 [hyper,1,95,88] P(i(i(x,a(y,z)),i(a(x,z),a(y,z)))).
200 [hyper,1,6,87] P(i(i(x,i(y,z)),i(x,i(i(u,z),i(a(y,u),z))))).
391 [hyper,1,136,3] P(i(x,a(y,a(z,x)))).
392 [hyper,1,136,2] P(i(x,a(y,a(x,z)))).
492 [hyper,1,164,5] P(i(i(x,y),i(i(z,x),i(z,y)))).
1161 [hyper,1,6,492] P(i(i(i(x,y),i(z,x)),i(i(x,y),i(z,y)))).
2535 [hyper,1,492,1161] P(i(i(x,i(i(y,z),i(u,y))),i(x,i(i(y,z),i(u,z))))).
5849 [hyper,1,2535,5] P(i(i(x,y),i(i(y,z),i(x,z)))).
10809 [hyper,1,200,89] P(i(x,i(i(y,a(z,u)),i(a(z,y),a(z,u))))).
4945776 [hyper,1,10809,10809] P(i(i(x,a(y,z)),i(a(y,x),a(y,z)))).
4946559 [hyper,1,4945776,392] P(i(a(x,y),a(x,a(y,z)))).
4946560 [hyper,1,4945776,391] P(i(a(x,y),a(x,a(z,y)))).
4962339 [hyper,1,188,4946559] P(i(a(a(x,y),a(y,z)),a(x,a(y,z)))).
4962659 [hyper,1,5849,4946560] P(i(i(a(x,a(y,z)),u),i(a(x,z),u))).
4968946 [hyper,1,4962659,4962339] P(i(a(a(x,y),z),a(x,a(y,z)))).

Proof of H321, .out99z21.

----> UNIT CONFLICT at 91.16 sec ----> 171586 [binary,171585.1,41.1] $ANS(H321).

Length of proof is 20. Level of proof is 8.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
2 [] P(i(x,a(x,y))).
3 [] P(i(x,a(y,x))).
4 [] P(i(i(x,y),i(i(z,y),i(a(x,z),y)))).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
41 [] -P(i(a(p,a(b,c)),a(a(p,b),c))) | $ANS(H321).
88 [hyper,1,5,3] P(i(x,i(y,a(z,y)))).
89 [hyper,1,5,2] P(i(x,i(y,a(y,z)))).
91 [hyper,1,5,6] P(i(x,i(i(y,i(z,u)),i(i(y,z),i(y,u))))).
95 [hyper,1,6,4] P(i(i(i(x,y),i(z,y)),i(i(x,y),i(a(x,z),y)))).
149 [hyper,1,6,91] P(i(i(x,i(y,i(z,u))),i(x,i(i(y,z),i(y,u))))).
232 [hyper,1,6,88] P(i(i(x,y),i(x,a(z,y)))).
270 [hyper,1,149,5] P(i(i(x,y),i(i(z,x),i(z,y)))).
279 [hyper,1,6,89] P(i(i(x,y),i(x,a(y,z)))).
410 [hyper,1,95,89] P(i(i(x,a(y,z)),i(a(x,y),a(y,z)))).
411 [hyper,1,95,88] P(i(i(x,a(y,z)),i(a(x,z),a(y,z)))).
455 [hyper,1,6,270] P(i(i(i(x,y),i(z,x)),i(i(x,y),i(z,y)))).
790 [hyper,1,270,455] P(i(i(x,i(i(y,z),i(u,y))),i(x,i(i(y,z),i(u,z))))).
11472 [hyper,1,232,2] P(i(x,a(y,a(x,z)))).
11882 [hyper,1,279,3] P(i(x,a(a(y,x),z))).
37272 [hyper,1,410,11472] P(i(a(x,y),a(y,a(x,z)))).
39000 [hyper,1,411,11882] P(i(a(x,y),a(a(z,x),y))).
79961 [hyper,1,790,5] P(i(i(x,y),i(i(y,z),i(x,z)))).
81318 [hyper,1,79961,37272] P(i(i(a(x,a(y,z)),u),i(a(y,x),u))).

86343 [hyper,1,410,39000] P(i(a(a(x,y),a(z,x)),a(a(z,x),y))).
171585 [hyper,1,81318,86343] P(i(a(x,a(y,z)),a(a(x,y),z))).

Proof of H322, .out99r.

----> UNIT CONFLICT at 0.21 sec ----> 2151 [binary,2150.1,42.1] $ANS(H322).

Length of proof is 4. Level of proof is 4.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
4 [] P(i(i(x,y),i(i(z,y),i(a(x,z),y)))).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
42 [] -P(i(a(p,p),p)) | $ANS(H322).
92 [hyper,1,6,5] P(i(i(x,y),i(x,x))).
162 [hyper,1,92,5] P(i(x,x)).
447 [hyper,1,4,162] P(i(i(x,y),i(a(y,x),y))).
2150 [hyper,1,447,162] P(i(a(x,x),x)).

Proof of H33, .out99z33.

----> UNIT CONFLICT at 845.31 sec ----> 591762 [binary,591761.1,43.1] $ANS(H33).

Length of proof is 24. Level of proof is 9.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
2 [] P(i(x,a(x,y))).
3 [] P(i(x,a(y,x))).
4 [] P(i(i(x,y),i(i(z,y),i(a(x,z),y)))).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
7 [] P(i(k(x,y),x)).
8 [] P(i(k(x,y),y)).
43 [] -P(i(k(i(p,b),i(c,d)),i(a(p,c),a(b,d)))) | $ANS(H33).
87 [hyper,1,5,4] P(i(x,i(i(y,z),i(i(u,z),i(a(y,u),z))))).
88 [hyper,1,5,3] P(i(x,i(y,a(z,y)))).
89 [hyper,1,5,2] P(i(x,i(y,a(y,z)))).
90 [hyper,1,6,6] P(i(i(i(x,i(y,z)),i(x,y)),i(i(x,i(y,z)),i(x,z)))).
91 [hyper,1,5,6] P(i(x,i(i(y,i(z,u)),i(i(y,z),i(y,u))))).
97 [hyper,1,5,7] P(i(x,i(k(y,z),y))).
102 [hyper,1,5,8] P(i(x,i(k(y,z),z))).
129 [hyper,1,6,91] P(i(i(x,i(y,i(z,u))),i(x,i(i(y,z),i(y,u))))).
146 [hyper,1,90,102] P(i(i(k(x,y),i(y,z)),i(k(x,y),z))).
217 [hyper,1,129,5] P(i(i(x,y),i(i(z,x),i(z,y)))).
243 [hyper,1,6,87] P(i(i(x,i(y,z)),i(x,i(i(u,z),i(a(y,u),z))))).
269 [hyper,1,6,88] P(i(i(x,y),i(x,a(z,y)))).
284 [hyper,1,6,217] P(i(i(i(x,y),i(z,x)),i(i(x,y),i(z,y)))).
298 [hyper,1,6,89] P(i(i(x,y),i(x,a(y,z)))).

425 [hyper,1,217,284] P(i(i(x,i(i(y,z),i(u,y))),i(x,i(i(y,z),i(u,z))))).
438 [hyper,1,284,97] P(i(i(x,y),i(k(x,z),y))).
4007 [hyper,1,217,146] P(i(i(x,i(k(y,z),i(z,u))),i(x,i(k(y,z),u)))).
4441 [hyper,1,425,5] P(i(i(x,y),i(i(y,z),i(x,z)))).
4577 [hyper,1,4441,269] P(i(i(i(x,a(y,z)),u),i(i(x,z),u))).
16570 [hyper,1,243,298] P(i(i(x,y),i(i(z,a(y,u)),i(a(x,z),a(y,u))))).
120637 [hyper,1,4441,16570] P(i(i(i(i(x,a(y,z)),i(a(u,x),a(y,z))),v),i(i(u,y),v))).
137839 [hyper,1,120637,4577] P(i(i(x,y),i(i(z,u),i(a(x,z),a(y,u))))).
591124 [hyper,1,4007,438] P(i(i(x,i(y,z)),i(k(x,y),z))).
591761 [hyper,1,591124,137839] P(i(k(i(x,y),i(z,u)),i(a(x,z),a(y,u)))).

Proof of H331, .out99z18.

----> UNIT CONFLICT at 25387.72 sec ----> 4183244 [binary,4183243.1,44.1] $ANS(H331).

Length of proof is 9. Level of proof is 6.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
2 [] P(i(x,a(x,y))).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
7 [] P(i(k(x,y),x)).
44 [] -P(i(i(p,b),i(k(p,c),a(b,d)))) | $ANS(H331).
95 [hyper,1,5,6] P(i(x,i(i(y,i(z,u)),i(i(y,z),i(y,u))))).
101 [hyper,1,5,7] P(i(x,i(k(y,z),y))).
153 [hyper,1,6,95] P(i(i(x,i(y,i(z,u))),i(x,i(i(y,z),i(y,u))))).
267 [hyper,1,153,5] P(i(i(x,y),i(i(z,x),i(z,y)))).
376 [hyper,1,6,267] P(i(i(i(x,y),i(z,x)),i(i(x,y),i(z,y)))).
390 [hyper,1,267,2] P(i(i(x,y),i(x,a(y,z)))).
8594 [hyper,1,267,390] P(i(i(x,i(y,z)),i(x,i(y,a(z,u))))).
39517 [hyper,1,376,101] P(i(i(x,y),i(k(x,z),y))).
4183243 [hyper,1,8594,39517] P(i(i(x,y),i(k(x,z),a(y,u)))).

Proof of H332, .out99r.

----> UNIT CONFLICT at 0.20 sec ----> 2092 [binary,2091.1,45.1] $ANS(H332).

Length of proof is 5. Level of proof is 4.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
4 [] P(i(i(x,y),i(i(z,y),i(a(x,z),y)))).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
45 [] -P(i(i(p,b),i(a(p,b),b))) | $ANS(H332).
92 [hyper,1,6,5] P(i(i(x,y),i(x,x))).
93 [hyper,1,6,4] P(i(i(i(x,y),i(z,y)),i(i(x,y),i(a(x,z),y)))).
162 [hyper,1,92,5] P(i(x,x)).
446 [hyper,1,5,162] P(i(x,i(y,y))).

2091 [hyper,1,93,446] P(i(i(x,y),i(a(x,y),y))).

Proof of H333, .out99z21.

----> UNIT CONFLICT at 0.10 sec ----> 810 [binary,809.1,46.1] $ANS(H333).

Length of proof is 6. Level of proof is 5.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
2 [] P(i(x,a(x,y))).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
46 [] -P(i(i(a(p,b),b),i(p,b))) | $ANS(H333).
89 [hyper,1,5,2] P(i(x,i(y,a(y,z)))).
91 [hyper,1,5,6] P(i(x,i(i(y,i(z,u)),i(i(y,z),i(y,u))))).
149 [hyper,1,6,91] P(i(i(x,i(y,i(z,u))),i(x,i(i(y,z),i(y,u))))).
270 [hyper,1,149,5] P(i(i(x,y),i(i(z,x),i(z,y)))).
455 [hyper,1,6,270] P(i(i(i(x,y),i(z,x)),i(i(x,y),i(z,y)))).
809 [hyper,1,455,89] P(i(i(a(x,y),z),i(x,z))).

Proof of H334, .out99z25.

----> UNIT CONFLICT at 20.01 sec ----> 82793 [binary,82792.1,47.1] $ANS(H334).

Length of proof is 8. Level of proof is 5.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
2 [] P(i(x,a(x,y))).
3 [] P(i(x,a(y,x))).
4 [] P(i(i(x,y),i(i(z,y),i(a(x,z),y)))).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
47 [] -P(i(i(p,b),i(a(p,c),a(b,c)))) | $ANS(H334).
88 [hyper,1,5,3] P(i(x,i(y,a(z,y)))).
89 [hyper,1,5,2] P(i(x,i(y,a(y,z)))).
95 [hyper,1,6,4] P(i(i(i(x,y),i(z,y)),i(i(x,y),i(a(x,z),y)))).
154 [hyper,1,6,89] P(i(i(x,y),i(x,a(y,z)))).
172 [hyper,1,95,88] P(i(i(x,a(y,z)),i(a(x,z),a(y,z)))).
487 [hyper,1,5,172] P(i(x,i(i(y,a(z,u)),i(a(y,u),a(z,u))))).
74679 [hyper,1,6,487] P(i(i(x,i(y,a(z,u))),i(x,i(a(y,u),a(z,u))))).
82792 [hyper,1,74679,154] P(i(i(x,y),i(a(x,z),a(y,z)))).

Proof of H335, .out99z21.

----> UNIT CONFLICT at 27.16 sec ----> 87795 [binary,87794.1,48.1] $ANS(H335).

Length of proof is 9. Level of proof is 5.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
2 [] P(i(x,a(x,y))).
3 [] P(i(x,a(y,x))).
4 [] P(i(i(x,y),i(i(z,y),i(a(x,z),y)))).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
48 [] -P(i(a(a(p,b),c),a(a(b,p),c))) | $ANS(H335).
88 [hyper,1,5,3] P(i(x,i(y,a(z,y)))).
89 [hyper,1,5,2] P(i(x,i(y,a(y,z)))).
95 [hyper,1,6,4] P(i(i(i(x,y),i(z,y)),i(i(x,y),i(a(x,z),y)))).
279 [hyper,1,6,89] P(i(i(x,y),i(x,a(y,z)))).
410 [hyper,1,95,89] P(i(i(x,a(y,z)),i(a(x,y),a(y,z)))).
411 [hyper,1,95,88] P(i(i(x,a(y,z)),i(a(x,z),a(y,z)))).
12020 [hyper,1,410,3] P(i(a(x,y),a(y,x))).
40344 [hyper,1,279,12020] P(i(a(x,y),a(a(y,x),z))).
87794 [hyper,1,411,40344] P(i(a(a(x,y),z),a(a(y,x),z))).

Proof of H3351, .out99z64.

----> UNIT CONFLICT at 961.60 sec ----> 597084 [binary,597083.1,49.1] $ANS(H3351).

Length of proof is 20. Level of proof is 10.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
2 [] P(i(x,a(x,y))).
3 [] P(i(x,a(y,x))).
4 [] P(i(i(x,y),i(i(z,y),i(a(x,z),y)))).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
49 [] -P(i(a(a(b,p),c),a(a(p,c),b))) | $ANS(H3351).
100 [hyper,1,5,6] P(i(x,i(i(y,i(z,u)),i(i(y,z),i(y,u))))).
103 [hyper,1,6,5] P(i(i(x,y),i(x,x))).
136 [hyper,1,6,100] P(i(i(x,i(y,i(z,u))),i(x,i(i(y,z),i(y,u))))).
187 [hyper,1,136,5] P(i(i(x,y),i(i(z,x),i(z,y)))).
215 [hyper,1,187,187] P(i(i(x,i(y,z)),i(x,i(i(u,y),i(u,z))))).
219 [hyper,1,6,187] P(i(i(i(x,y),i(z,x)),i(i(x,y),i(z,y)))).
231 [hyper,1,187,2] P(i(i(x,y),i(x,a(y,z)))).
265 [hyper,1,215,4] P(i(i(x,y),i(i(z,i(u,y)),i(z,i(a(x,u),y))))).
267 [hyper,1,187,219] P(i(i(x,i(i(y,z),i(u,y))),i(x,i(i(y,z),i(u,z))))).
396 [hyper,1,267,5] P(i(i(x,y),i(i(y,z),i(x,z)))).
676 [hyper,1,103,5] P(i(x,x)).
37626 [hyper,1,265,3] P(i(i(x,i(y,a(z,u))),i(x,i(a(u,y),a(z,u))))).
37627 [hyper,1,265,2] P(i(i(x,i(y,a(z,u))),i(x,i(a(z,y),a(z,u))))).
54537 [hyper,1,37626,231] P(i(i(x,y),i(a(z,x),a(y,z)))).
55307 [hyper,1,37627,676] P(i(i(x,a(y,z)),i(a(y,x),a(y,z)))).
74090 [hyper,1,54537,3] P(i(a(x,y),a(a(z,y),x))).
74091 [hyper,1,54537,2] P(i(a(x,y),a(a(y,z),x))).
332493 [hyper,1,396,74090] P(i(i(a(a(x,y),z),u),i(a(z,y),u))).

332677 [hyper,1,55307,74091] P(i(a(a(x,y),a(z,x)),a(a(x,y),z))).
597083 [hyper,1,332493,332677] P(i(a(a(x,y),z),a(a(y,z),x))).

Proof of H34, .out99z33.

----> UNIT CONFLICT at 8878.79 sec ----> 2515548 [binary,2515547.1,50.1] $ANS(H34).

Length of proof is 12. Level of proof is 7.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
2 [] P(i(x,a(x,y))).
3 [] P(i(x,a(y,x))).
4 [] P(i(i(x,y),i(i(z,y),i(a(x,z),y)))).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
7 [] P(i(k(x,y),x)).
8 [] P(i(k(x,y),y)).
9 [] P(i(i(x,y),i(i(x,z),i(x,k(y,z))))).
50 [] -P(i(a(k(p,c),k(b,c)),k(a(p,b),c))) | $ANS(H34).
88 [hyper,1,5,3] P(i(x,i(y,a(z,y)))).
89 [hyper,1,5,2] P(i(x,i(y,a(y,z)))).
269 [hyper,1,6,88] P(i(i(x,y),i(x,a(z,y)))).
298 [hyper,1,6,89] P(i(i(x,y),i(x,a(y,z)))).
4162 [hyper,1,269,7] P(i(k(x,y),a(z,x))).
4423 [hyper,1,298,7] P(i(k(x,y),a(x,z))).
6696 [hyper,1,9,4162] P(i(i(k(x,y),z),i(k(x,y),k(a(u,x),z)))).
6782 [hyper,1,9,4423] P(i(i(k(x,y),z),i(k(x,y),k(a(x,u),z)))).
2340600 [hyper,1,6696,8] P(i(k(x,y),k(a(z,x),y))).
2376619 [hyper,1,6782,8] P(i(k(x,y),k(a(x,z),y))).
2395387 [hyper,1,4,2376619] P(i(i(x,k(a(y,z),u)),i(a(k(y,u),x),k(a(y,z),u)))).
2515547 [hyper,1,2395387,2340600] P(i(a(k(x,y),k(z,y)),k(a(x,z),y))).

Proof of H341, .out99z64.

----> UNIT CONFLICT at 44.97 sec ----> 109793 [binary,109792.1,51.1] $ANS(H341).

Length of proof is 23. Level of proof is 11.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
2 [] P(i(x,a(x,y))).
3 [] P(i(x,a(y,x))).
4 [] P(i(i(x,y),i(i(z,y),i(a(x,z),y)))).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
7 [] P(i(k(x,y),x)).
8 [] P(i(k(x,y),y)).
9 [] P(i(i(x,y),i(i(x,z),i(x,k(y,z))))).
51 [] -P(i(k(a(p,b),c),a(k(p,c),k(b,c)))) | $ANS(H341).

100 [hyper,1,5,6] P(i(x,i(i(y,i(z,u)),i(i(y,z),i(y,u))))).
103 [hyper,1,6,5] P(i(i(x,y),i(x,x))).
136 [hyper,1,6,100] P(i(i(x,i(y,i(z,u))),i(x,i(i(y,z),i(y,u))))).
187 [hyper,1,136,5] P(i(i(x,y),i(i(z,x),i(z,y)))).
215 [hyper,1,187,187] P(i(i(x,i(y,z)),i(x,i(i(u,y),i(u,z))))).
219 [hyper,1,6,187] P(i(i(i(x,y),i(z,x)),i(i(x,y),i(z,y)))).
231 [hyper,1,187,2] P(i(i(x,y),i(x,a(y,z)))).
265 [hyper,1,215,4] P(i(i(x,y),i(i(z,i(u,y)),i(z,i(a(x,u),y))))).
267 [hyper,1,187,219] P(i(i(x,i(i(y,z),i(u,y))),i(x,i(i(y,z),i(u,z))))).
396 [hyper,1,267,5] P(i(i(x,y),i(i(y,z),i(x,z)))).
579 [hyper,1,396,8] P(i(i(x,y),i(k(z,x),y))).
676 [hyper,1,103,5] P(i(x,x)).
960 [hyper,1,136,579] P(i(i(x,i(y,z)),i(i(k(u,x),y),i(k(u,x),z)))).
5273 [hyper,1,9,676] P(i(i(x,y),i(x,k(x,y)))).
9120 [hyper,1,187,5273] P(i(i(x,i(y,z)),i(x,i(y,k(y,z))))).
19383 [hyper,1,9120,5] P(i(x,i(y,k(y,x)))).
31683 [hyper,1,396,19383] P(i(i(i(x,k(x,y)),z),i(y,z))).
37626 [hyper,1,265,3] P(i(i(x,i(y,a(z,u))),i(x,i(a(u,y),a(z,u))))).
54537 [hyper,1,37626,231] P(i(i(x,y),i(a(z,x),a(y,z)))).
73599 [hyper,1,31683,54537] P(i(x,i(a(y,z),a(k(z,x),y)))).
84642 [hyper,1,960,73599] P(i(i(k(x,y),a(z,u)),i(k(x,y),a(k(u,y),z)))).
100252 [hyper,1,84642,7] P(i(k(a(x,y),z),a(k(y,z),x))).
109792 [hyper,1,84642,100252] P(i(k(a(x,y),z),a(k(x,z),k(y,z)))).

Proof of H3421, .out99z33.

----> UNIT CONFLICT at 7157.89 sec ----> 2379678 [binary,2379677.1,52.1] $ANS(H3421).

Length of proof is 10. Level of proof is 7.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
2 [] P(i(x,a(x,y))).
3 [] P(i(x,a(y,x))).
4 [] P(i(i(x,y),i(i(z,y),i(a(x,z),y)))).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
7 [] P(i(k(x,y),x)).
8 [] P(i(k(x,y),y)).
9 [] P(i(i(x,y),i(i(x,z),i(x,k(y,z))))).
52 [] -P(i(a(k(p,b),c),k(a(p,c),a(b,c)))) | $ANS(H3421).
89 [hyper,1,5,2] P(i(x,i(y,a(y,z)))).
298 [hyper,1,6,89] P(i(i(x,y),i(x,a(y,z)))).
4422 [hyper,1,298,8] P(i(k(x,y),a(y,z))).
4423 [hyper,1,298,7] P(i(k(x,y),a(x,z))).
6745 [hyper,1,4,4422] P(i(i(x,a(y,z)),i(a(k(u,y),x),a(y,z)))).
6784 [hyper,1,4,4423] P(i(i(x,a(y,z)),i(a(k(y,u),x),a(y,z)))).
2366266 [hyper,1,6745,3] P(i(a(k(x,y),z),a(y,z))).
2377721 [hyper,1,6784,3] P(i(a(k(x,y),z),a(x,z))).
2378320 [hyper,1,9,2377721] P(i(i(a(k(x,y),z),u),i(a(k(x,y),z),k(a(x,z),u)))).
2379677 [hyper,1,2378320,2366266] P(i(a(k(x,y),z),k(a(x,z),a(y,z)))).

Proof of H3422, .out99z51.

----> UNIT CONFLICT at 231.74 sec ----> 281538 [binary,281537.1,53.1] $ANS(H3422).

Length of proof is 26. Level of proof is 12.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
2 [] P(i(x,a(x,y))).
3 [] P(i(x,a(y,x))).
4 [] P(i(i(x,y),i(i(z,y),i(a(x,z),y)))).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
7 [] P(i(k(x,y),x)).
8 [] P(i(k(x,y),y)).
9 [] P(i(i(x,y),i(i(x,z),i(x,k(y,z))))).
53 [] -P(i(k(a(p,c),a(b,c)),a(k(p,b),c))) | $ANS(H3422).
95 [hyper,1,5,3] P(i(x,i(y,a(z,y)))).
97 [hyper,1,5,6] P(i(x,i(i(y,i(z,u)),i(i(y,z),i(y,u))))).
100 [hyper,1,6,5] P(i(i(x,y),i(x,x))).
101 [hyper,1,6,4] P(i(i(i(x,y),i(z,y)),i(i(x,y),i(a(x,z),y)))).
111 [hyper,1,6,9] P(i(i(i(x,y),i(x,z)),i(i(x,y),i(x,k(y,z))))).
139 [hyper,1,6,97] P(i(i(x,i(y,i(z,u))),i(x,i(i(y,z),i(y,u))))).
163 [hyper,1,101,95] P(i(i(x,a(y,z)),i(a(x,z),a(y,z)))).
185 [hyper,1,111,100] P(i(i(x,y),i(x,k(y,x)))).
232 [hyper,1,139,5] P(i(i(x,y),i(i(z,x),i(z,y)))).
281 [hyper,1,6,232] P(i(i(i(x,y),i(z,x)),i(i(x,y),i(z,y)))).
285 [hyper,1,232,185] P(i(i(x,i(y,z)),i(x,i(y,k(z,y))))).
286 [hyper,1,232,163] P(i(i(x,i(y,a(z,u))),i(x,i(a(y,u),a(z,u))))).
296 [hyper,1,232,2] P(i(i(x,y),i(x,a(y,z)))).
304 [hyper,1,232,281] P(i(i(x,i(i(y,z),i(u,y))),i(x,i(i(y,z),i(u,z))))).
345 [hyper,1,285,5] P(i(x,i(y,k(x,y)))).
364 [hyper,1,286,296] P(i(i(x,y),i(a(x,z),a(y,z)))).
410 [hyper,1,304,5] P(i(i(x,y),i(i(y,z),i(x,z)))).
517 [hyper,1,232,364] P(i(i(x,i(y,z)),i(x,i(a(y,u),a(z,u))))).
622 [hyper,1,410,8] P(i(i(x,y),i(k(z,x),y))).
759 [hyper,1,517,345] P(i(x,i(a(y,z),a(k(x,y),z)))).
1070 [hyper,1,139,622] P(i(i(x,i(y,z)),i(i(k(u,x),y),i(k(u,x),z)))).
1602 [hyper,1,1070,759] P(i(i(k(x,y),a(z,u)),i(k(x,y),a(k(y,z),u)))).
2350 [hyper,1,1602,7] P(i(k(a(x,y),z),a(k(z,x),y))).
2782 [hyper,1,410,2350] P(i(i(a(k(x,y),z),u),i(k(a(y,z),x),u))).
2789 [hyper,1,163,2350] P(i(a(k(a(x,y),z),y),a(k(z,x),y))).
281537 [hyper,1,2782,2789] P(i(k(a(x,y),a(z,y)),a(k(x,z),y))).

Proof of H35, .out99z70u.

----> UNIT CONFLICT at 695.22 sec ----> 232810 [binary,232809.1,24.1] $ANS(H35).

Length of proof is 25. Level of proof is 13.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
4 [] P(i(x,i(y,x))).
5 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
6 [] P(i(k(x,y),x)).
7 [] P(i(k(x,y),y)).
9 [] P(i(x,a(x,y))).
10 [] P(i(x,a(y,x))).
11 [] P(i(i(x,y),i(i(z,y),i(a(x,z),y)))).
24 [] -P(i(k(k(i(p,a(b,c)),i(b,d)),i(c,e)),i(p,a(d,e)))) | $ANS(H35).
42 [hyper,1,4,5] P(i(x,i(i(y,i(z,u)),i(i(y,z),i(y,u))))).
47 [hyper,1,4,6] P(i(x,i(k(y,z),y))).
87 [hyper,1,11,10] P(i(i(x,a(y,z)),i(a(z,x),a(y,z)))).
99 [hyper,1,5,42] P(i(i(x,i(y,i(z,u))),i(x,i(i(y,z),i(y,u))))).
189 [hyper,1,99,4] P(i(i(x,y),i(i(z,x),i(z,y)))).
270 [hyper,1,189,189] P(i(i(x,i(y,z)),i(x,i(i(u,y),i(u,z))))).
278 [hyper,1,5,189] P(i(i(i(x,y),i(z,x)),i(i(x,y),i(z,y)))).
285 [hyper,1,189,87] P(i(i(x,i(y,a(z,u))),i(x,i(a(u,y),a(z,u))))).
290 [hyper,1,189,9] P(i(i(x,y),i(x,a(y,z)))).
294 [hyper,1,189,278] P(i(i(x,i(i(y,z),i(u,y))),i(x,i(i(y,z),i(u,z))))).
420 [hyper,1,285,290] P(i(i(x,y),i(a(z,x),a(y,z)))).
504 [hyper,1,294,4] P(i(i(x,y),i(i(y,z),i(x,z)))).
934 [hyper,1,504,504] P(i(i(i(i(x,y),i(z,y)),u),i(i(z,x),u))).
959 [hyper,1,504,420] P(i(i(i(a(x,y),a(z,x)),u),i(i(y,z),u))).
994 [hyper,1,504,7] P(i(i(x,y),i(k(z,x),y))).
1713 [hyper,1,99,994] P(i(i(x,i(y,z)),i(i(k(u,x),y),i(k(u,x),z)))).
2576 [hyper,1,5,1713] P(i(i(i(x,i(y,z)),i(k(u,x),y)),i(i(x,i(y,z)),i(k(u,x),z)))).
8067 [hyper,1,189,270] P(i(i(x,i(y,i(z,u))),i(x,i(y,i(i(v,z),i(v,u)))))).
13355 [hyper,1,934,959] P(i(i(x,a(y,z)),i(i(z,u),i(x,a(u,y))))).
43951 [hyper,1,959,13355] P(i(i(x,y),i(i(z,u),i(a(z,x),a(u,y))))).
106774 [hyper,1,8067,43951] P(i(i(x,y),i(i(z,u),i(i(v,a(z,x)),i(v,a(u,y)))))).
231987 [hyper,1,2576,47] P(i(i(x,i(y,z)),i(k(y,x),z))).
232019 [hyper,1,189,231987] P(i(i(x,i(y,i(z,u))),i(x,i(k(z,y),u)))).
232386 [hyper,1,232019,232019] P(i(i(x,i(y,i(z,u))),i(k(k(z,y),x),u))).
232809 [hyper,1,232386,106774] P(i(k(k(i(x,a(y,z)),i(y,u)),i(z,v)),i(x,a(u,v)))).

Proof of H351, .out99z67.

----> UNIT CONFLICT at 19.54 sec ----> 61676 [binary,61675.1,56.1] $ANS(H351).

Length of proof is 27. Level of proof is 12.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
4 [] P(i(i(x,y),i(i(z,y),i(a(x,z),y)))).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
7 [] P(i(k(x,y),x)).
8 [] P(i(k(x,y),y)).
9 [] P(i(i(x,y),i(i(x,z),i(x,k(y,z))))).
56 [] -P(i(k(k(i(p,a(b,c)),i(b,d)),i(c,d)),i(p,d))) | $ANS(H351).
99 [hyper,1,6,6] P(i(i(i(x,i(y,z)),i(x,y)),i(i(x,i(y,z)),i(x,z)))).

100 [hyper,1,5,6] P(i(x,i(i(y,i(z,u)),i(i(y,z),i(y,u))))).
106 [hyper,1,5,7] P(i(x,i(k(y,z),y))).
118 [hyper,1,9,8] P(i(i(k(x,y),z),i(k(x,y),k(y,z)))).
136 [hyper,1,6,100] P(i(i(x,i(y,i(z,u))),i(x,i(i(y,z),i(y,u))))).
140 [hyper,1,99,106] P(i(i(k(x,y),i(x,z)),i(k(x,y),z))).
194 [hyper,1,136,5] P(i(i(x,y),i(i(z,x),i(z,y)))).
226 [hyper,1,6,194] P(i(i(i(x,y),i(z,x)),i(i(x,y),i(z,y)))).
232 [hyper,1,194,8] P(i(i(x,k(y,z)),i(x,z))).
334 [hyper,1,194,226] P(i(i(x,i(i(y,z),i(u,y))),i(x,i(i(y,z),i(u,z))))).
1962 [hyper,1,140,106] P(i(k(k(x,y),z),x)).
2034 [hyper,1,232,7] P(i(k(k(x,y),z),y)).
2043 [hyper,1,334,5] P(i(i(x,y),i(i(y,z),i(x,z)))).
2045 [hyper,1,2043,2043] P(i(i(i(i(x,y),i(z,y)),u),i(i(z,x),u))).
2124 [hyper,1,2043,8] P(i(i(x,y),i(k(z,x),y))).
2128 [hyper,1,2045,2045] P(i(i(x,i(y,z)),i(i(u,y),i(x,i(u,z))))).
2446 [hyper,1,2043,2124] P(i(i(i(k(x,y),z),u),i(i(y,z),u))).
2534 [hyper,1,2124,4] P(i(k(x,i(y,z)),i(i(u,z),i(a(y,u),z)))).
6739 [hyper,1,118,2034] P(i(k(k(x,y),z),k(z,y))).
7542 [hyper,1,2446,140] P(i(i(x,i(y,z)),i(k(y,x),z))).
7569 [hyper,1,2128,7542] P(i(i(x,k(y,z)),i(i(z,i(y,u)),i(x,u)))).
7941 [hyper,1,140,2534] P(i(k(i(x,y),i(z,y)),i(a(z,x),y))).
14290 [hyper,1,9,6739] P(i(i(k(k(x,y),z),u),i(k(k(x,y),z),k(k(z,y),u)))).
18607 [hyper,1,2128,7941] P(i(i(x,a(y,z)),i(k(i(z,u),i(y,u)),i(x,u)))).
29782 [hyper,1,14290,1962] P(i(k(k(x,y),z),k(k(z,y),x))).
47284 [hyper,1,7569,29782] P(i(i(x,i(k(y,z),u)),i(k(k(x,z),y),u))).
61675 [hyper,1,47284,18607] P(i(k(k(i(x,a(y,z)),i(y,u)),i(z,u)),i(x,u))).

Proof of H36, .out99z10.

----> UNIT CONFLICT at   6.24 sec ----> 19949 [binary,19948.1,57.1] $ANS(H36).

Length of proof is 10. Level of proof is 7.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
4 [] P(i(i(x,y),i(i(z,y),i(a(x,z),y)))).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
57 [] -P(i(a(p,b),i(i(p,b),b))) | $ANS(H36).
87 [hyper,1,5,5] P(i(x,i(y,i(z,y)))).
94 [hyper,1,6,6] P(i(i(i(x,i(y,z)),i(x,y)),i(i(x,i(y,z)),i(x,z)))).
97 [hyper,1,6,4] P(i(i(i(x,y),i(z,y)),i(i(x,y),i(a(x,z),y)))).
144 [hyper,1,94,87] P(i(i(x,i(i(y,x),z)),i(x,z))).
148 [hyper,1,97,87] P(i(i(x,i(y,z)),i(a(x,z),i(y,z)))).
205 [hyper,1,144,5] P(i(x,x)).
251 [hyper,1,6,205] P(i(i(i(x,y),x),i(i(x,y),y))).
362 [hyper,1,5,251] P(i(x,i(i(i(y,z),y),i(i(y,z),z)))).
9326 [hyper,1,144,362] P(i(x,i(i(x,y),y))).
19948 [hyper,1,148,9326] P(i(a(x,y),i(i(x,y),y))).

Proof of H42, .out99z21.

----> UNIT CONFLICT at 0.51 sec ----> 4495 [binary,4494.1,58.1] $ANS(H42).

Length of proof is 8. Level of proof is 6.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
10 [] P(i(i(x,n(y)),i(y,n(x)))).
58 [] -P(i(i(p,b),i(n(b),n(p)))) | $ANS(H42).
94 [hyper,1,6,5] P(i(i(x,y),i(x,x))).
117 [hyper,1,5,10] P(i(x,i(i(y,n(z)),i(z,n(y))))).
216 [hyper,1,6,117] P(i(i(x,i(y,n(z))),i(x,i(z,n(y))))).
362 [hyper,1,94,5] P(i(x,x)).
714 [hyper,1,10,362] P(i(x,n(n(x)))).
1333 [hyper,1,5,714] P(i(x,i(y,n(n(y))))).
3161 [hyper,1,6,1333] P(i(i(x,y),i(x,n(n(y))))).
4494 [hyper,1,216,3161] P(i(i(x,y),i(n(y),n(x)))).

Proof of H422, .out99z66.

----> UNIT CONFLICT at 11202.79 sec ----> 2186878 [binary,2186877.1,59.1] $ANS(H422).

Length of proof is 11. Level of proof is 7.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
10 [] P(i(i(x,n(y)),i(y,n(x)))).
59 [] -P(i(i(p,b),i(n(n(p)),n(n(b))))) | $ANS(H422).
100 [hyper,1,5,6] P(i(x,i(i(y,i(z,u)),i(i(y,z),i(y,u))))).
103 [hyper,1,6,5] P(i(i(x,y),i(x,x))).
143 [hyper,1,6,100] P(i(i(x,i(y,i(z,u))),i(x,i(i(y,z),i(y,u))))).
225 [hyper,1,143,5] P(i(i(x,y),i(i(z,x),i(z,y)))).
253 [hyper,1,103,5] P(i(x,x)).
306 [hyper,1,225,10] P(i(i(x,i(y,n(z))),i(x,i(z,n(y))))).
2977 [hyper,1,10,253] P(i(x,n(n(x)))).
12764 [hyper,1,225,2977] P(i(i(x,y),i(x,n(n(y))))).
35260 [hyper,1,306,12764] P(i(i(x,y),i(n(y),n(x)))).
1706731 [hyper,1,225,35260] P(i(i(x,i(y,z)),i(x,i(n(z),n(y))))).
2186877 [hyper,1,1706731,35260] P(i(i(x,y),i(n(n(x)),n(n(y))))).

Proof of H423, .out99z61.

----> UNIT CONFLICT at 0.16 sec ----> 1405 [binary,1404.1,60.1] $ANS(H423).

Length of proof is 29. Level of proof is 15.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
7 [] P(i(k(x,y),x)).
8 [] P(i(k(x,y),y)).
9 [] P(i(i(x,y),i(i(x,z),i(x,k(y,z))))).
10 [] P(i(i(x,n(y)),i(y,n(x)))).
11 [] P(i(n(x),i(x,y))).
60 [] -P(i(i(k(p,b),c),i(k(p,n(c)),n(b)))) | $ANS(H423).
91 [hyper,1,5,5] P(i(x,i(y,i(z,y)))).
97 [hyper,1,6,6] P(i(i(i(x,i(y,z)),i(x,y)),i(i(x,i(y,z)),i(x,z)))).
98 [hyper,1,5,6] P(i(x,i(i(y,i(z,u)),i(i(y,z),i(y,u))))).
140 [hyper,1,97,91] P(i(i(x,i(i(y,x),z)),i(x,z))).
141 [hyper,1,6,98] P(i(i(x,i(y,i(z,u))),i(x,i(i(y,z),i(y,u))))).
174 [hyper,1,140,5] P(i(x,x)).
189 [hyper,1,141,5] P(i(i(x,y),i(i(z,x),i(z,y)))).
217 [hyper,1,9,174] P(i(i(x,y),i(x,k(x,y)))).
229 [hyper,1,5,189] P(i(x,i(i(y,z),i(i(u,y),i(u,z))))).
235 [hyper,1,189,10] P(i(i(x,i(y,n(z))),i(x,i(z,n(y))))).
255 [hyper,1,5,217] P(i(x,i(i(y,z),i(y,k(y,z))))).
281 [hyper,1,141,229] P(i(x,i(i(i(y,z),i(u,y)),i(i(y,z),i(u,z))))).
332 [hyper,1,140,255] P(i(x,i(y,k(y,x)))).
340 [hyper,1,140,281] P(i(i(x,y),i(i(y,z),i(x,z)))).
440 [hyper,1,340,332] P(i(i(i(x,k(x,y)),z),i(y,z))).
449 [hyper,1,340,189] P(i(i(i(i(x,y),i(x,z)),u),i(i(y,z),u))).
459 [hyper,1,340,8] P(i(i(x,y),i(k(z,x),y))).
460 [hyper,1,340,7] P(i(i(x,y),i(k(x,z),y))).
617 [hyper,1,449,440] P(i(i(k(x,y),z),i(y,i(x,z)))).
624 [hyper,1,449,235] P(i(i(x,n(y)),i(i(z,x),i(y,n(z))))).
673 [hyper,1,459,11] P(i(k(x,n(y)),i(y,z))).
687 [hyper,1,340,460] P(i(i(i(k(x,y),z),u),i(i(x,z),u))).
921 [hyper,1,6,673] P(i(i(k(x,n(y)),y),i(k(x,n(y)),z))).
999 [hyper,1,687,921] P(i(i(x,y),i(k(x,n(y)),z))).
1092 [hyper,1,235,999] P(i(i(x,y),i(z,n(k(x,n(y)))))).
1183 [hyper,1,140,1092] P(i(i(x,y),n(k(x,n(y))))).
1247 [hyper,1,624,1183] P(i(i(x,i(y,z)),i(k(y,n(z)),n(x)))).
1337 [hyper,1,189,1247] P(i(i(x,i(y,i(z,u))),i(x,i(k(z,n(u)),n(y))))).
1404 [hyper,1,1337,617] P(i(i(k(x,y),z),i(k(x,n(z)),n(y)))).

Proof of H424, .out99z5.

----> UNIT CONFLICT at 161587.87 sec ----> 4338552 [binary,4338551.1,61.1] $ANS(H424).

Length of proof is 14. Level of proof is 10.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).

7 [] P(i(k(x,y),x)).
8 [] P(i(k(x,y),y)).
10 [] P(i(i(x,n(y)),i(y,n(x)))).
11 [] P(i(n(x),i(x,y))).
61 [] -P(i(k(i(p,b),n(b)),n(p))) | $ANS(H424).
96 [hyper,1,6,6] P(i(i(i(x,i(y,z)),i(x,y)),i(i(x,i(y,z)),i(x,z)))).
97 [hyper,1,5,6] P(i(x,i(i(y,i(z,u)),i(i(y,z),i(y,u))))).
99 [hyper,1,5,7] P(i(x,i(k(y,z),y))).
133 [hyper,1,6,97] P(i(i(x,i(y,i(z,u))),i(x,i(i(y,z),i(y,u))))).
134 [hyper,1,96,99] P(i(i(k(x,y),i(x,z)),i(k(x,y),z))).
182 [hyper,1,133,5] P(i(i(x,y),i(i(z,x),i(z,y)))).
229 [hyper,1,182,182] P(i(i(x,i(y,z)),i(x,i(i(u,y),i(u,z))))).
236 [hyper,1,182,10] P(i(i(x,i(y,n(z))),i(x,i(z,n(y))))).
317 [hyper,1,229,11] P(i(n(x),i(i(y,x),i(y,z)))).
24879 [hyper,1,182,317] P(i(i(x,n(y)),i(x,i(i(z,y),i(z,u))))).
2301773 [hyper,1,24879,8] P(i(k(x,n(y)),i(i(z,y),i(z,u)))).
4338432 [hyper,1,134,2301773] P(i(k(i(x,y),n(y)),i(x,z))).
4338516 [hyper,1,236,4338432] P(i(k(i(x,y),n(y)),i(z,n(x)))).
4338551 [hyper,1,134,4338516] P(i(k(i(x,y),n(y)),n(x))).

Proof of H43, .out99r.

----> UNIT CONFLICT at 0.05 sec ----> 443 [binary,441.1,62.1] $ANS(H43).

Length of proof is 3. Level of proof is 3.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
10 [] P(i(i(x,n(y)),i(y,n(x)))).
62 [] -P(i(p,n(n(p)))) | $ANS(H43).
92 [hyper,1,6,5] P(i(i(x,y),i(x,x))).
162 [hyper,1,92,5] P(i(x,x)).
441 [hyper,1,10,162] P(i(x,n(n(x)))).

Proof of H431, .out99r.

----> UNIT CONFLICT at 0.05 sec ----> 442 [binary,441.1,63.1] $ANS(H431).

Length of proof is 3. Level of proof is 3.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
10 [] P(i(i(x,n(y)),i(y,n(x)))).
63 [] -P(i(n(p),n(n(n(p))))) | $ANS(H431).
92 [hyper,1,6,5] P(i(i(x,y),i(x,x))).
162 [hyper,1,92,5] P(i(x,x)).

441 [hyper,1,10,162] P(i(x,n(n(x)))).

Proof of H432, .out99r.

----> UNIT CONFLICT at 201.44 sec ----> 216642 [binary,216641.1,64.1] $ANS(H432).

Length of proof is 7. Level of proof is 7.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
10 [] P(i(i(x,n(y)),i(y,n(x)))).
64 [] -P(i(n(n(n(p))),n(p))) | $ANS(H432).
92 [hyper,1,6,5] P(i(i(x,y),i(x,x))).
162 [hyper,1,92,5] P(i(x,x)).
441 [hyper,1,10,162] P(i(x,n(n(x)))).
1896 [hyper,1,5,441] P(i(x,i(y,n(n(y))))).
65506 [hyper,1,6,1896] P(i(i(x,y),i(x,n(n(y))))).
210350 [hyper,1,65506,441] P(i(x,n(n(n(n(x)))))).
216641 [hyper,1,10,210350] P(i(n(n(n(x))),n(x))).

Proof of H44, .out99r.

----> UNIT CONFLICT at 59.55 sec ----> 93092 [binary,93091.1,65.1] $ANS(H44).

Length of proof is 5. Level of proof is 5.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
7 [] P(i(k(x,y),x)).
8 [] P(i(k(x,y),y)).
11 [] P(i(n(x),i(x,y))).
65 [] -P(i(k(p,n(p)),b)) | $ANS(H44).
118 [hyper,1,5,11] P(i(x,i(n(y),i(y,z)))).
873 [hyper,1,6,118] P(i(i(x,n(y)),i(x,i(y,z)))).
41261 [hyper,1,873,8] P(i(k(x,n(y)),i(y,z))).
92966 [hyper,1,6,41261] P(i(i(k(x,n(y)),y),i(k(x,n(y)),z))).
93091 [hyper,1,92966,7] P(i(k(x,n(x)),y)).

Proof of H441, .out99r.

----> UNIT CONFLICT at 21768.48 sec ----> 2168847 [binary,2168846.1,66.1] $ANS(H441).

Length of proof is 9. Level of proof is 7.

---------------- PROOF ----------------

1 [] -P(i(x,y))| -P(x)|P(y).
4 [] P(i(i(x,y),i(i(z,y),i(a(x,z),y)))).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
7 [] P(i(k(x,y),x)).
8 [] P(i(k(x,y),y)).
11 [] P(i(n(x),i(x,y))).
66 [] -P(i(a(k(p,n(p)),b),b))|$ANS(H441).
92 [hyper,1,6,5] P(i(i(x,y),i(x,x))).
118 [hyper,1,5,11] P(i(x,i(n(y),i(y,z)))).
162 [hyper,1,92,5] P(i(x,x)).
873 [hyper,1,6,118] P(i(i(x,n(y)),i(x,i(y,z)))).
41261 [hyper,1,873,8] P(i(k(x,n(y)),i(y,z))).
92966 [hyper,1,6,41261] P(i(i(k(x,n(y)),y),i(k(x,n(y)),z))).
93091 [hyper,1,92966,7] P(i(k(x,n(x)),y)).
93369 [hyper,1,4,93091] P(i(i(x,y),i(a(k(z,n(z)),x),y))).
2168846 [hyper,1,93369,162] P(i(a(k(x,n(x)),y),y)).

Proof of H442, .out99r.

----> UNIT CONFLICT at 123754.47 sec ----> 4203360 [binary,4203359.1,67.1] $ANS(H442).

Length of proof is 15. Level of proof is 9.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
4 [] P(i(i(x,y),i(i(z,y),i(a(x,z),y)))).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
7 [] P(i(k(x,y),x)).
8 [] P(i(k(x,y),y)).
11 [] P(i(n(x),i(x,y))).
67 [] -P(i(k(a(p,b),n(p)),b)) | $ANS(H442).
83 [hyper,1,5,5] P(i(x,i(y,i(z,y)))).
90 [hyper,1,6,6] P(i(i(i(x,i(y,z)),i(x,y)),i(i(x,i(y,z)),i(x,z)))).
93 [hyper,1,6,4] P(i(i(i(x,y),i(z,y)),i(i(x,y),i(a(x,z),y)))).
99 [hyper,1,5,8] P(i(x,i(k(y,z),z))).
117 [hyper,1,6,11] P(i(i(n(x),x),i(n(x),y))).
167 [hyper,1,93,83] P(i(i(x,i(y,z)),i(a(x,z),i(y,z)))).
184 [hyper,1,90,99] P(i(i(k(x,y),i(y,z)),i(k(x,y),z))).
805 [hyper,1,5,117] P(i(x,i(i(n(y),y),i(n(y),z)))).
37897 [hyper,1,6,805] P(i(i(x,i(n(y),y)),i(x,i(n(y),z)))).
1402238 [hyper,1,37897,5] P(i(x,i(n(x),y))).
1403006 [hyper,1,167,1402238] P(i(a(x,y),i(n(x),y))).
1412672 [hyper,1,5,1403006] P(i(x,i(a(y,z),i(n(y),z)))).
1438160 [hyper,1,6,1412672] P(i(i(x,a(y,z)),i(x,i(n(y),z)))).
4202613 [hyper,1,1438160,7] P(i(k(a(x,y),z),i(n(x),y))).
4203359 [hyper,1,184,4202613] P(i(k(a(x,y),n(x)),y)).

Proof of H443, .out99z43.

----> UNIT CONFLICT at 48696.89 sec ----> 3844788 [binary,3844787.1,68.1] $ANS(H443).

Length of proof is 12. Level of proof is 6.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
4 [] P(i(i(x,y),i(i(z,y),i(a(x,z),y)))).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
10 [] P(i(i(x,n(y)),i(y,n(x)))).
11 [] P(i(n(x),i(x,y))).
68 [] -P(i(n(p),i(n(b),n(a(p,b))))) | $ANS(H443).
94 [hyper,1,5,4] P(i(x,i(i(y,z),i(i(u,z),i(a(y,u),z))))).
101 [hyper,1,6,5] P(i(i(x,y),i(x,x))).
127 [hyper,1,5,10] P(i(x,i(i(y,n(z)),i(z,n(y))))).
219 [hyper,1,6,94] P(i(i(x,i(y,z)),i(x,i(i(u,z),i(a(y,u),z))))).
266 [hyper,1,101,5] P(i(x,x)).
522 [hyper,1,6,127] P(i(i(x,i(y,n(z))),i(x,i(z,n(y))))).
985 [hyper,1,219,11] P(i(n(x),i(i(y,z),i(a(x,y),z)))).
1317 [hyper,1,10,266] P(i(x,n(n(x)))).
2957 [hyper,1,6,985] P(i(i(n(x),i(y,z)),i(n(x),i(a(x,y),z)))).
4682 [hyper,1,5,1317] P(i(x,i(y,n(n(y))))).
3805401 [hyper,1,2957,4682] P(i(n(x),i(a(x,y),n(n(y))))).
3844787 [hyper,1,522,3805401] P(i(n(x),i(n(y),n(a(x,y))))).

Proof of H4441, .out99z21.

----> UNIT CONFLICT at 1092.19 sec ----> 654212 [binary,654211.1,69.1] $ANS(H4441).

Length of proof is 11. Level of proof is 9.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
2 [] P(i(x,a(x,y))).
3 [] P(i(x,a(y,x))).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
9 [] P(i(i(x,y),i(i(x,z),i(x,k(y,z))))).
10 [] P(i(i(x,n(y)),i(y,n(x)))).
69 [] -P(i(n(a(p,b)),k(n(p),n(b)))) | $ANS(H4441).
94 [hyper,1,6,5] P(i(i(x,y),i(x,x))).
362 [hyper,1,94,5] P(i(x,x)).
714 [hyper,1,10,362] P(i(x,n(n(x)))).
1333 [hyper,1,5,714] P(i(x,i(y,n(n(y))))).
3161 [hyper,1,6,1333] P(i(i(x,y),i(x,n(n(y))))).
4609 [hyper,1,3161,3] P(i(x,n(n(a(y,x))))).
4610 [hyper,1,3161,2] P(i(x,n(n(a(x,y))))).
646014 [hyper,1,10,4609] P(i(n(a(x,y)),n(y))).
650930 [hyper,1,10,4610] P(i(n(a(x,y)),n(x))).
653592 [hyper,1,9,650930] P(i(i(n(a(x,y)),z),i(n(a(x,y)),k(n(x),z)))).

654211 [hyper,1,653592,646014] P(i(n(a(x,y)),k(n(x),n(y)))).

Proof of H4442, .out99z19.

----> UNIT CONFLICT at 23957.58 sec ----> 4057515 [binary,4057514.1,70.1] $ANS(H4442).

Length of proof is 5. Level of proof is 4.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
4 [] P(i(i(x,y),i(i(z,y),i(a(x,z),y)))).
7 [] P(i(k(x,y),x)).
8 [] P(i(k(x,y),y)).
10 [] P(i(i(x,n(y)),i(y,n(x)))).
70 [] -P(i(k(n(p),n(b)),n(a(p,b)))) | $ANS(H4442).
121 [hyper,1,10,8] P(i(x,n(k(y,n(x))))).
122 [hyper,1,10,7] P(i(x,n(k(n(x),y)))).
2817 [hyper,1,4,122] P(i(i(x,n(k(n(y),z))),i(a(y,x),n(k(n(y),z))))).
621952 [hyper,1,2817,121] P(i(a(x,y),n(k(n(x),n(y))))).
4057514 [hyper,1,10,621952] P(i(k(n(x),n(y)),n(a(x,y)))).

Proof of H445, .out99r.

----> UNIT CONFLICT at 6773.35 sec ----> 1403027 [binary,1403026.1,72.1] $ANS(H445).

Length of proof is 6. Level of proof is 5.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
4 [] P(i(i(x,y),i(i(z,y),i(a(x,z),y)))).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
11 [] P(i(n(x),i(x,y))).
72 [] -P(i(a(p,n(p)),i(n(n(p)),p))) | $ANS(H445).
84 [hyper,1,4,5] P(i(i(x,i(y,z)),i(a(z,x),i(y,z)))).
117 [hyper,1,6,11] P(i(i(n(x),x),i(n(x),y))).
805 [hyper,1,5,117] P(i(x,i(i(n(y),y),i(n(y),z)))).
37897 [hyper,1,6,805] P(i(i(x,i(n(y),y)),i(x,i(n(y),z)))).
1402238 [hyper,1,37897,5] P(i(x,i(n(x),y))).
1403026 [hyper,1,84,1402238] P(i(a(x,y),i(n(y),x))).

Proof of H446, .out99r.

----> UNIT CONFLICT at 0.03 sec ----> 245 [binary,244.1,73.1] $ANS(H446).

Length of proof is 2. Level of proof is 2.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
4 [] P(i(i(x,y),i(i(z,y),i(a(x,z),y)))).
5 [] P(i(x,i(y,x))).
11 [] P(i(n(x),i(x,y))).
73 [] -P(i(a(n(p),b),i(p,b))) | $ANS(H446).
119 [hyper,1,4,11] P(i(i(x,i(y,z)),i(a(n(y),x),i(y,z)))).
244 [hyper,1,119,5] P(i(a(n(x),y),i(x,y))).

Proof of H447, .out99r.

----> UNIT CONFLICT at 6773.35 sec ----> 1403007 [binary,1403006.1,74.1] $ANS(H447).

Length of proof is 8. Level of proof is 5.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
4 [] P(i(i(x,y),i(i(z,y),i(a(x,z),y)))).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
11 [] P(i(n(x),i(x,y))).
74 [] -P(i(a(p,b),i(n(p),b))) | $ANS(H447).
83 [hyper,1,5,5] P(i(x,i(y,i(z,y)))).
93 [hyper,1,6,4] P(i(i(i(x,y),i(z,y)),i(i(x,y),i(a(x,z),y)))).
117 [hyper,1,6,11] P(i(i(n(x),x),i(n(x),y))).
167 [hyper,1,93,83] P(i(i(x,i(y,z)),i(a(x,z),i(y,z)))).
805 [hyper,1,5,117] P(i(x,i(i(n(y),y),i(n(y),z)))).
37897 [hyper,1,6,805] P(i(i(x,i(n(y),y)),i(x,i(n(y),z)))).
1402238 [hyper,1,37897,5] P(i(x,i(n(x),y))).
1403006 [hyper,1,167,1402238] P(i(a(x,y),i(n(x),y))).

Proof of HIF1, .out99z70w.

----> UNIT CONFLICT at 24318.68 sec ----> 1847947 [binary,1847946.1,14.1] $ANS(HIF1).

Length of proof is 19. Level of proof is 11.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
4 [] P(i(x,i(y,x))).
5 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
6 [] P(i(k(x,y),x)).
7 [] P(i(k(x,y),y)).
8 [] P(i(i(x,y),i(i(x,z),i(x,k(y,z))))).
14 [] -P(k(i(k(i(p,q),i(p,r)),i(p,k(q,r))),i(i(p,k(q,r)),k(i(p,q),i(p,r))))) | $ANS(HIF1).
31 [hyper,1,4,5] P(i(x,i(i(y,i(z,u)),i(i(y,z),i(y,u))))).
32 [hyper,1,5,4] P(i(i(x,y),i(x,x))).
36 [hyper,1,4,6] P(i(x,i(k(y,z),y))).
41 [hyper,1,5,8] P(i(i(i(x,y),i(x,z)),i(i(x,y),i(x,k(y,z))))).
88 [hyper,1,5,31] P(i(i(x,i(y,i(z,u))),i(x,i(i(y,z),i(y,u))))).
109 [hyper,1,41,32] P(i(i(x,y),i(x,k(y,x)))).

178 [hyper,1,88,4] P(i(i(x,y),i(i(z,x),i(z,y)))).
267 [hyper,1,5,178] P(i(i(i(x,y),i(z,x)),i(i(x,y),i(z,y)))).
281 [hyper,1,178,7] P(i(i(x,k(y,z)),i(x,z))).
1060 [hyper,1,267,36] P(i(i(x,y),i(k(x,z),y))).
1074 [hyper,1,5,36] P(i(i(x,k(y,z)),i(x,y))).
1855 [hyper,1,88,1060] P(i(i(x,i(y,z)),i(i(k(x,u),y),i(k(x,u),z)))).
2726 [hyper,1,1855,8] P(i(i(k(i(x,y),z),i(x,u)),i(k(i(x,y),z),i(x,k(y,u))))).
7397 [hyper,1,8,1074] P(i(i(i(x,k(y,z)),u),i(i(x,k(y,z)),k(i(x,y),u)))).
7654 [hyper,1,2726,7] P(i(k(i(x,y),i(x,z)),i(x,k(y,z)))).
11126 [hyper,1,7397,281] P(i(i(x,k(y,z)),k(i(x,y),i(x,z)))).
13457 [hyper,1,4,7654] P(i(x,i(k(i(y,z),i(y,u)),i(y,k(z,u))))).
1847936 [hyper,1,109,13457] P(i(x,k(i(k(i(y,z),i(y,u)),i(y,k(z,u))),x))).
1847946 [hyper,1,1847936,11126] P(k(i(k(i(x,y),i(x,z)),i(x,k(y,z))),i(i(u,k(v,w)),k(i(u,v),i(u,w))))).

Proof of HIF2, .out99z33.

----> UNIT CONFLICT at 845.15 sec ----> 591208 [binary,591207.1,76.1] $ANS(HIF2).

Length of proof is 27. Level of proof is 11.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
7 [] P(i(k(x,y),x)).
8 [] P(i(k(x,y),y)).
9 [] P(i(i(x,y),i(i(x,z),i(x,k(y,z))))).
76 [] -P(k(i(i(p,i(q,r)),i(k(p,q),r)),i(i(k(p,q),r),i(p,i(q,r))))) | $ANS(HIF2).
90 [hyper,1,6,6] P(i(i(i(x,i(y,z)),i(x,y)),i(i(x,i(y,z)),i(x,z)))).
91 [hyper,1,5,6] P(i(x,i(i(y,i(z,u)),i(i(y,z),i(y,u))))).
94 [hyper,1,6,5] P(i(i(x,y),i(x,x))).
97 [hyper,1,5,7] P(i(x,i(k(y,z),y))).
102 [hyper,1,5,8] P(i(x,i(k(y,z),z))).
106 [hyper,1,6,9] P(i(i(i(x,y),i(x,z)),i(i(x,y),i(x,k(y,z))))).
129 [hyper,1,6,91] P(i(i(x,i(y,i(z,u))),i(x,i(i(y,z),i(y,u))))).
146 [hyper,1,90,102] P(i(i(k(x,y),i(y,z)),i(k(x,y),z))).
217 [hyper,1,129,5] P(i(i(x,y),i(i(z,x),i(z,y)))).
284 [hyper,1,6,217] P(i(i(i(x,y),i(z,x)),i(i(x,y),i(z,y)))).
285 [hyper,1,5,217] P(i(x,i(i(y,z),i(i(u,y),i(u,z))))).
438 [hyper,1,284,97] P(i(i(x,y),i(k(x,z),y))).
463 [hyper,1,94,5] P(i(x,x)).
465 [hyper,1,284,285] P(i(i(i(i(x,y),i(x,z)),u),i(i(y,z),u))).
1165 [hyper,1,106,94] P(i(i(x,y),i(x,k(y,x)))).
4007 [hyper,1,217,146] P(i(i(x,i(k(y,z),i(z,u))),i(x,i(k(y,z),u)))).
5261 [hyper,1,9,463] P(i(i(x,y),i(x,k(x,y)))).
5488 [hyper,1,5,1165] P(i(x,i(i(y,z),i(y,k(z,y))))).
5782 [hyper,1,6,5488] P(i(i(x,i(y,z)),i(x,i(y,k(z,y))))).
5885 [hyper,1,5782,5] P(i(x,i(y,k(x,y)))).
10667 [hyper,1,5,5885] P(i(x,i(y,i(z,k(y,z))))).
261907 [hyper,1,284,10667] P(i(i(i(x,k(y,x)),z),i(y,z))).
262091 [hyper,1,465,261907] P(i(i(k(x,y),z),i(x,i(y,z)))).

262651 [hyper,1,5,262091] P(i(x,i(i(k(y,z),u),i(y,i(z,u))))).
262847 [hyper,1,5261,262651] P(i(x,k(x,i(i(k(y,z),u),i(y,i(z,u)))))).
591124 [hyper,1,4007,438] P(i(i(x,i(y,z)),i(k(x,y),z))).
591207 [hyper,1,262847,591124] P(k(i(i(x,i(y,z)),i(k(x,y),z)),i(i(k(u,v),w),i(u,i(v,w))))).

Proof of HIF3, .out99z9.

----> UNIT CONFLICT at 838187.01 sec ----> 6343963 [binary,6343962.1,77.1] $ANS(HIF3).

Length of proof is 13. Level of proof is 10.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
9 [] P(i(i(x,y),i(i(x,z),i(x,k(y,z))))).
77 [] -P(k(i(i(p,i(q,r)),i(q,i(p,r))),i(i(q,i(p,r)),i(p,i(q,r))))) | $ANS(HIF3).
87 [hyper,1,5,5] P(i(x,i(y,i(z,y)))).
94 [hyper,1,6,6] P(i(i(i(x,i(y,z)),i(x,y)),i(i(x,i(y,z)),i(x,z)))).
144 [hyper,1,94,87] P(i(i(x,i(i(y,x),z)),i(x,z))).
198 [hyper,1,5,144] P(i(x,i(i(y,i(i(z,y),u)),i(y,u)))).
205 [hyper,1,144,5] P(i(x,x)).
250 [hyper,1,9,205] P(i(i(x,y),i(x,k(x,y)))).
7322 [hyper,1,144,198] P(i(i(i(x,y),z),i(y,z))).
18675 [hyper,1,5,7322] P(i(x,i(i(i(y,z),u),i(z,u)))).
1034182 [hyper,1,6,18675] P(i(i(x,i(i(y,z),u)),i(x,i(z,u)))).
6340744 [hyper,1,1034182,6] P(i(i(x,i(y,z)),i(y,i(x,z)))).
6340777 [hyper,1,5,6340744] P(i(x,i(i(y,i(z,u)),i(z,i(y,u))))).
6343888 [hyper,1,250,6340777] P(i(x,k(x,i(i(y,i(z,u)),i(z,i(y,u)))))).
6343962 [hyper,1,6343888,6340744] P(k(i(i(x,i(y,z)),i(y,i(x,z))),i(i(u,i(v,w)),i(v,i(u,w))))).

Proof of HIF4, .out99z55k.

----> UNIT CONFLICT at   0.22 sec ----> 1644 [binary,1643.1,12.1] $ANS(HIF4).

Length of proof is 29. Level of proof is 13.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
2 [] P(i(x,a(x,y))).
3 [] P(i(x,a(y,x))).
4 [] P(i(i(x,y),i(i(z,y),i(a(x,z),y)))).
7 [] P(i(x,i(y,x))).
8 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
9 [] P(i(k(x,y),x)).
10 [] P(i(k(x,y),y)).
11 [] P(i(i(x,y),i(i(x,z),i(x,k(y,z))))).
12 [] -P(k(i(a(k(p,q),r),k(a(p,r),a(q,r))),i(k(a(p,r),a(q,r)),a(k(p,q),r)))) | $ANS(HIF4).
31 [hyper,1,7,3] P(i(x,i(y,a(z,y)))).
33 [hyper,1,7,8] P(i(x,i(i(y,i(z,u)),i(i(y,z),i(y,u))))).

36 [hyper,1,8,7] P(i(i(x,y),i(x,x))).
39 [hyper,1,8,4] P(i(i(i(x,y),i(z,y)),i(i(x,y),i(a(x,z),y)))).
51 [hyper,1,8,11] P(i(i(i(x,y),i(x,z)),i(i(x,y),i(x,k(y,z))))).
72 [hyper,1,8,33] P(i(i(x,i(y,i(z,u))),i(x,i(i(y,z),i(y,u))))).
88 [hyper,1,39,31] P(i(i(x,a(y,z)),i(a(x,z),a(y,z)))).
110 [hyper,1,51,36] P(i(i(x,y),i(x,k(y,x)))).
153 [hyper,1,72,7] P(i(i(x,y),i(i(z,x),i(z,y)))).
205 [hyper,1,153,110] P(i(i(x,i(y,z)),i(x,i(y,k(z,y))))).
206 [hyper,1,153,88] P(i(i(x,i(y,a(z,u))),i(x,i(a(y,u),a(z,u))))).
216 [hyper,1,153,2] P(i(i(x,y),i(x,a(y,z)))).
241 [hyper,1,205,7] P(i(x,i(y,k(x,y)))).
261 [hyper,1,206,216] P(i(i(x,y),i(a(x,z),a(y,z)))).
314 [hyper,1,153,241] P(i(i(x,y),i(x,i(z,k(y,z))))).
391 [hyper,1,153,261] P(i(i(x,i(y,z)),i(x,i(a(y,u),a(z,u))))).
412 [hyper,1,261,10] P(i(a(k(x,y),z),a(y,z))).
413 [hyper,1,261,9] P(i(a(k(x,y),z),a(x,z))).
471 [hyper,1,314,10] P(i(k(x,y),i(z,k(y,z)))).
562 [hyper,1,72,391] P(i(i(x,i(y,z)),i(i(x,a(y,u)),i(x,a(z,u))))).
630 [hyper,1,11,413] P(i(i(a(k(x,y),z),u),i(a(k(x,y),z),k(a(x,z),u)))).
743 [hyper,1,562,471] P(i(i(k(x,y),a(z,u)),i(k(x,y),a(k(y,z),u)))).
791 [hyper,1,630,412] P(i(a(k(x,y),z),k(a(x,z),a(y,z)))).
893 [hyper,1,743,9] P(i(k(a(x,y),z),a(k(z,x),y))).
1002 [hyper,1,241,791] P(i(x,k(i(a(k(y,z),u),k(a(y,u),a(z,u))),x))).
1134 [hyper,1,88,893] P(i(a(k(a(x,y),z),y),a(k(z,x),y))).
1460 [hyper,1,153,1134] P(i(i(x,a(k(a(y,z),u),z)),i(x,a(k(u,y),z)))).
1578 [hyper,1,1460,893] P(i(k(a(x,y),a(z,y)),a(k(x,z),y))).
1643 [hyper,1,1002,1578] P(k(i(a(k(x,y),z),k(a(x,z),a(y,z))),i(k(a(u,v),a(w,v)),a(k(u,w),v)))).

Proof of HIF5, .out99z21.

----> UNIT CONFLICT at 16194.44 sec ----> 3580014 [binary,3580013.1,79.1] $ANS(HIF5).

Length of proof is 20. Level of proof is 10.

---------------- PROOF ----------------

1 [] -P(i(x,y)) | -P(x) | P(y).
2 [] P(i(x,a(x,y))).
3 [] P(i(x,a(y,x))).
4 [] P(i(i(x,y),i(i(z,y),i(a(x,z),y)))).
5 [] P(i(x,i(y,x))).
6 [] P(i(i(x,i(y,z)),i(i(x,y),i(x,z)))).
7 [] P(i(k(x,y),x)).
8 [] P(i(k(x,y),y)).
9 [] P(i(i(x,y),i(i(x,z),i(x,k(y,z))))).
10 [] P(i(i(x,n(y)),i(y,n(x)))).
79 [] -P(k(i(n(a(p,q)),k(n(p),n(q))),i(k(n(p),n(q)),n(a(p,q))))) | $ANS(HIF5).
94 [hyper,1,6,5] P(i(i(x,y),i(x,x))).
121 [hyper,1,10,8] P(i(x,n(k(y,n(x))))).
122 [hyper,1,10,7] P(i(x,n(k(n(x),y)))).
362 [hyper,1,94,5] P(i(x,x)).
714 [hyper,1,10,362] P(i(x,n(n(x)))).

717 [hyper,1,9,362] P(i(i(x,y),i(x,k(x,y)))).
1333 [hyper,1,5,714] P(i(x,i(y,n(n(y))))).
2334 [hyper,1,4,122] P(i(i(x,n(k(n(y),z))),i(a(y,x),n(k(n(y),z))))).
3161 [hyper,1,6,1333] P(i(i(x,y),i(x,n(n(y))))).
4609 [hyper,1,3161,3] P(i(x,n(n(a(y,x))))).
4610 [hyper,1,3161,2] P(i(x,n(n(a(x,y))))).
412775 [hyper,1,2334,121] P(i(a(x,y),n(k(n(x),n(y))))).
646014 [hyper,1,10,4609] P(i(n(a(x,y)),n(y))).
650930 [hyper,1,10,4610] P(i(n(a(x,y)),n(x))).
653592 [hyper,1,9,650930] P(i(i(n(a(x,y)),z),i(n(a(x,y)),k(n(x),z)))).
654211 [hyper,1,653592,646014] P(i(n(a(x,y)),k(n(x),n(y)))).
3577037 [hyper,1,10,412775] P(i(k(n(x),n(y)),n(a(x,y)))).
3578433 [hyper,1,5,3577037] P(i(x,i(k(n(y),n(z)),n(a(y,z))))).
3579333 [hyper,1,717,3578433] P(i(x,k(x,i(k(n(y),n(z)),n(a(y,z)))))).
3580013 [hyper,1,3579333,654211] P(k(i(n(a(x,y)),k(n(x),n(y))),i(k(n(z),n(u)),n(a(z,u))))).

You now have access to sixty-three so-called short proofs. In addition to the proofs themselves, the accompanying statistics, such as time to complete and number of clauses required, provide you with abundant food for thought, that time-honored aphorism. Such data can be used both to evaluate some new idea or strategy you might formulate and to stimulate you to formulate a new approach. Perhaps an analysis of which of the ten Horn axioms was used for which proof merits some contemplation. The citation for each proof concerning its origin, which output file the proof was taken from, indicates how hard it was to find the result—the name suggests how many experiments were relied upon.

8. Acknowledgments and Promises

If you have derived enjoyment and excitement from what is written here, you can thank my colleague Michael Beeson. Indeed, he was the main motivating force for the research reported here. I certainly thank him. He has throughout the years found interesting areas for me to study. I believe, as a promise, he will find others in the future.

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Gail Pieper is a demanding reader. Her standards are in part reflected in these notebooks, but, of course, I can only approximate what she seeks.

Judy Beumer facilitates access to my webiste writings. Without her, you would not find the notebooks.

I hope that we will meet again, perhaps in a later notebook or perhaps as you visit or revisit earlier-written notebooks. At this point, I know not what will come next. I assume, almost promise, that more will come and that we will meet in future narratives.